Specialized Discounted Cash Flow Analysis Formulas for Valuation of Benefits and Costs of Urban Trees and Forests

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Abstract

Discounted cash flow (DCF) analysis is a method of valuation often used in forests managed for timber production objectives to obtain the present value (PV) of cash flows, or the value in current day dollars considering interest (Bullard and Straka 1998). Several conventional forestry valuation software packages use DCF as a method for financial decision-making because it accounts for the time value of money and represents the dynamic financial nature of a timber stand. Early forest valuation models, such as Faustman’s formula, rely on the principles of DCF analysis to determine important forestry investment financial criteria such as land expectation value (LEV) and financial optima like rotation length (Tietenberg and Lewis 2008). DCF analysis produces reliable monetary valuations for natural resources, including forests (Gollier et al. 2008). DCF is often used over long time spans with good results; however, its use to value long life assets, like trees, may produce issues like fairness to future generations and inflation estimates.

Despite its accepted use in forestry for timber production, DCF analysis, or the income approach generally, has not been frequently used in urban forestry and arboriculture. Cash flows for benefits and costs from single trees or urban forests are difficult to determine, and the mathematical structure of DCF analysis is somewhat complicated (Council of Tree and Landscape Appraisers 2000; Straka and Bullard 2006). Negative cash flows or expenditures (both capital and operating) are called “costs” in traditional forestry investment analysis, but they are more likely to be labeled as “expenses” in an appraisal income approach.

Conventional forestry valuation software packages (such as FORVAL) can be used for DCF calculations, but they require that cash flows be input in one of a few standard structures (single sum, terminating annuity, perpetual annuity, or perpetual periodic series) (Straka and Bullard 2002). These standard structures have rigid assumptions about the cash flow sequences; for example, a cash flow occurring each year and beginning at year 1 or a cash flow occurring periodically every x years and beginning at year x (Straka and Bullard 2002). Benefits (i.e., income) and costs in urban forest and tree valuation situations do not always occur in these structured patterns and standard DCF formulas do not handle irregular cash flows well. This is another primary reason the income approach is often difficult to apply in these situations (Bullard and Straka 2006).

We identified a series of specialized discounting formulas that were well-suited for solving valuation problems that follow typical cash flow patterns occurring in the benefit and cost structures of urban tree and forest situations; that is, those that do not follow standard structured cash flow patterns and, thus, would be difficult to value using many conventional DCF formulas (McPherson and Simpson 2002; McPherson 2003; McPherson 2007). Using the standard DCF formulas common to forest valuation (Appraisal Institute 2008, Bullard and Straka 1998) as a foundation, we constructed a series of new or “special” DCF formulas that will allow these benefit and cost situations to be evaluated using conventional DCF valuation software packages. We also reviewed the basic standard DCF formulas as they are the basis of the “specialized” formulas. Most formulas could be utilized as part of a standard DCF valuation software models like FORVAL (Straka and Bullard 2006; Bullard et al. 2011).

**Single-sum Discounting (SSD)**

The basic formula used in DCF analysis is the formula for discounting a single sum. It discounts a cash flow to year zero on a cash flow time line. Year zero represents the current point in time or the beginning of year one or time period one. This formula is:

\[
V_0 = \frac{V_n}{(1 + i)^n}
\]

Where \( V_0 \) is the value at year zero, \( V_n \) is the value at year \( n \), \( i \) is the interest rate (expressed as a decimal), and \( n \) is the number of years being evaluated.

**Present Value of a Terminating Annuity (TA)**

Sometimes, cash flows of the same magnitude occur annually. The basic formula calculates the present value of a terminating annual series as:

\[
V_0 = a \left( \frac{(1 + i)^n - 1}{i(1 + i)^n} \right)
\]

Where \( a \) is the annual cash flow and the remaining variables are as defined previously.

**Present Value of a Perpetual Annuity (PA)**

In some urban forestry situations (such as the creation of a conservation easement that generates perpetual uniform benefits over time), the value of an annual cash flow occurs forever. The calculation of a perpetual annuity is simply:

\[
V_0 = \frac{a}{i}
\]

Where \( a \) is the annual cash flow and the remaining variables are as defined previously.
Present Value of a Terminating Periodic Series (TPS)

The prior valuation formulas were basic DCF analysis tools. Most valuation software packages include an automatic computation of these values. The TPS formula is not a basic DCF formula. Terminating periodic refers to a situation where benefits or costs have a regular, uniform magnitude, but occur on a periodic, not an annual basis. An example would be stormwater or flood mitigation every 20 years, starting at year 20 and ending at year 140. The formula could easily be adapted to time periods shorter than a year. The TPS formula is:

\[ V_0 = a \frac{(1+i)^n - 1}{(1+i)^n - 1} \]

Where \( t \) is the length of each period in years, \( n \) is the number of compounding periods, and the remaining variables are as defined previously.

Present Value of a Fixed Rate Increasing Annuity (FRIA)

In other situations, the benefits or costs may occur annually, but have a magnitude that increases at an exponential rate. For example, a tree’s ability to sequester carbon may increase a given rate per year. In this case, we can use a formula for the present value of a growing annuity. The calculation of the FRIA is:

\[ V_0 = \frac{a}{(1+g)} \left[ 1 - \left( \frac{1+g}{1+i} \right)^n \right] \]

Where \( g \) is the percentage rate of growth of the annuity (expressed as a decimal) and the remaining variables are as defined previously.

Present Value of Minimum Size Delayed Annual Cash Flows (MSDACF)

In some urban trees, annual cash flows may not occur until the tree reaches a certain minimum size. For example, electricity savings in summer from tree shade or privacy benefits from a large tree do not begin until the tree reaches a certain size. Other examples might be privacy benefits, sound barrier benefits, air quality, health, and recreation benefits (Martin 1989; Novak et al. 2002; Wolf 2004). In fact, MSDACF valuation is common in urban forestry applications, as many urban forest benefits rely on a certain crown size or structure more than a particular age or diameter breast height (DBH). These crown assets only occur once the tree has reached a minimum age for developing a mature crown. The MSDACF formula is:

\[ V_0 = a \frac{(1+i)^{na} - 1}{i(1+i)^{na}(1+i)^{nv}} \]

Where \( na \) is the number of years for which the annuity occurs and \( nv \) is the number of years the annuity is delayed from the standard annuity. We note that this formula also applies to costs with similar financial scheduling, like periodic costs for pruning.
Present Value of Minimum Size Delayed Periodic Cash Flows (MSDPCF)

Similar to the MSDACF, the MSDPCF calculates the present value of benefits (or costs) incurred periodically that are contingent upon the tree reaching a certain “minimum size.” An example would be the “windbreak” ability of a tree in a windstorm. First, the tree would need to reach a minimum size to have windbreak ability and, second, the benefit would occur periodically, not every year. The MSDPCF formula is:

\[
V_0 = a \frac{(1 + i)^{nt} - 1}{(1 + i)^t - 1(1 + i)^{nt} (1 + i)^{nv}}
\]

Where \(n_{at}\) is the number of years for which the series occurs, \(t\) is the length of the time period, and \(n_v\) is the number of years in the future the series begins.

Present Value of Patterned Terminating Periodic Series (PTPS)

Urban trees may have several systematic, “stacked” cash flows, where one cash flow is “stacked” onto another. A cash flow of a smaller magnitude (i.e., the base series) may occur on a frequent basis, but necessitate a cash flow of a larger magnitude (i.e., the stacked series) on an infrequent basis. An example would be the soil enhancement benefit of trees. Fertilization might be reduced on an annual basis (the base series) and soil aeration might be reduced every 10 years (i.e., the stacked series). In this case, the larger cash flow is stacked onto the pattern of the smaller cash flow, and the following formula should be used:

\[
V_0 = a_1 \frac{(1 + i)^{n_{a1}t} - 1}{(1 + i)^{n_{a1}t} - 1(1 + i)^{n_{a2}t} (1 + i)^{n_{a2}t}} - (a_2 - a_1) \frac{(1 + i)^{n_{a2}t} - 1}{(1 + i)^{n_{a1}t} - 1(1 + i)^{n_{a2}t} (1 + i)^{n_{a2}t}}
\]

Where \(a_1\) is the cash flow of the base series, \(a_2\) is the cash flow of the stacked series, \(i\) is the interest rate, \(n_1\) is number of years the base series occurs, \(t_1\) is length of the time period for the base series, and \(n_2\) is number of years the stacked series occurs, and \(t_2\) is length of the time period for the stacked series.

Present Value of Minimum Size Delayed Patterned Terminating Cash Flows (MSDPTCF)

Like other benefits or costs that do not begin until a minimum tree size occurs, patterned terminating benefits or costs need be discounted back to year zero. A systematic pruning of a tree on two levels is an example of this calculation; for example, minor pruning every five years and major pruning every twenty years. If so, the following formula should be used:

\[
V_0 = a_1 \frac{(1 + i)^{n_{a1}t} - 1}{(1 + i)^{n_{a1}t} - 1(1 + i)^{n_{a2}t} (1 + i)^{n_{a2}t}} - (a_2 - a_1) \frac{(1 + i)^{n_{a2}t} - 1}{(1 + i)^{n_{a1}t} - 1(1 + i)^{n_{a2}t} (1 + i)^{n_{a2}t}}
\]

Where \(a_1\) is the cash flow of the base series, \(a_2\) is the cash flow of the stacked series, \(n_1\) is number of years for which the base series occurs, \(t_1\) is length of the time period for the base series, and \(n_2\) is number of years the stacked series occurs, \(t_2\) is length of the time period for the stacked series, \(n_{a1}\) is number of years the base annuity is away from year zero, and \(n_{a2}\) is number of years the stacked annuity is away from year zero.
Urban Tree Site Value (UTSV)

In the traditional forestry literature LEV or bare land value is calculated for land in permanent timber production (Klemperer 1996). This methodology can be used to calculate the PV of any perpetual cash flow-producing investment (Straka and Bullard 1996). This means a site value for an urban tree can also be calculated by compounding the PV of the tree’s cash flows to the end of its “rotation” (defined as its viable life on the site) and assessing this over a perpetual time frame. The following formula accomplishes this:

\[
UTSV = \frac{PV (1 + i)^n}{(1 + i)^n - 1}
\]

\(UTSV\) is the urban tree site value with a perpetual time horizon, while \(PV\) is the present value of all benefits and costs of the tree for one “rotation,” and \(n\) is the length of the “rotation.”

Literature Cited