SELECTING REASONABLE (REALISTIC) DISCOUNT RATES: A DIFFERENT PERSPECTIVE OF VALUE INCREASE RATES... AND TIME

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Part I, Introduction of the Concept

In economics the term discount rate or interest rate has many meanings. To some it means the social time value of money; to others, the cost of borrowing or holding money. When used to evaluate investments, the discount rate is often referred to as the alternative rate of return. That is, the rate of return which could be earned if the investable funds were put into the best available alternative. It is this latter meaning that is the basis of this paper, and which is critical to its development.

All too often analysts are told or encouraged to choose the discount rate or alternative rate of return..., as if it were strictly a matter of choice..., a matter of what the analyst would like to make on his/her investment. The proper instruction ought to be to determine the discount rate; to consider what rates are available in the market..., to determine what specific rate is reasonable and realistically attainable in light of such factors as risk, degree of liquidity, and, all too often ignored, the duration of the investment period.

Factors such as risk and liquidity will not be considered in this presentation, other than indirectly. It is investment duration and its influence on the alternative rate of return (discount rate) that will be the main subject of exploration.

Often, the determination of a discount rate follows a rationale such as: If investable funds can be invested in, say the money market, than the money market rate shall be the rate for evaluating potential investments. If a potential investment has a duration of a few months, the money market rate will likely be a realistic and reasonably accurate discount rate. However, if the potential investment has a duration of, say, 10 years, it is questionable that the money market rate will be appropriate because of the wide variations that that rate will have during a 10-year period. A more reasonable and realistic discount rate would be one based on a 10-year investment period or instrument, such as a 10-year C.D. It is this rate-of-value-growth/duration-length relationship that this paper will explore, first with intuitive reasoning and logic, and then with some empirical evidence.

Reasonable value growth rates and investment periods.

Defining regions of feasibility is a common approach in developing general concepts. Therefore, this approach will be used to determine if there are feasible regions (reasonable combinations) of compound growth rates and durations, and infeasible (unreasonable) ones. Let figure 1 represent all possible combinations of compound growth rates and durations (time).** Let us consider the feasibility of each of the indicated rate/time combinations in turn --

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** Combinations of negative growth rates and time are included for the sake of balance. They are not germane to the concept being developed.
Points | Logical conclusions
--- | ---
A (and -A): Extremely high rates (and negative rates) of growth over extremely short periods of time: ... Feasible (possible)
B (and -B): Extremely low rates (and negative rates) of growth over extremely short periods of time: ... Feasible (possible)
C (and -C): Extremely low rates (and negative rates) of growth over extremely long periods of time: ... Feasible (possible)
D (and -D): Extremely high rates (and negative rates) of growth over extremely long periods of time: ... Infeasible (impossible)

The infeasibility of combination D (and -D) needs emphasis. Nothing in the physical Universe can expand at a constant high compound rate for very long. Growth, and therefore the rate of growth, is controlled by the familiar law of diminishing returns. Holding all other factors constant while varying time will most certainly cause the rate of growth to eventually suffer diminishing returns. Compound rates of increase, unfettered by reasonable time limits, are phenomena of mathematics and not of reality. According to accepted theory, the Universe expanded radially at the speed of light following the "Big Bang." During the first earth-year following the Bang, the Universe expanded in volume at a near-infinite rate, from a relatively small volume to over 4 cubic light-years. During the second earth-year, its rate of volume increase dropped to some 700%. By the 50th earth-year its rate of volume increase had dropped to less than 7%. By the 1,000th earth-year, its rate of expansion was only 0.3%. Our Universe is still expanding radially at the speed of light, however its current volume increase rate is infinitesimal (4x10^-7%).

Thus, with point D (and -D) being infeasible and points A, B and C (and their negative counter-parts) being feasible, the feasible and infeasible regions must be separated by a left to right, downward sloping line (upward sloping in the negative rate quadrant). Faced with this conclusion, two questions must be answered: What is the shape of these demarkation lines? and, Where are they located?
The answer to the second question is critical to the concept being developed. If the line is located so far up and to the right (hereafter I'll reference primarily the positive rate portion of the graph) that the infeasible region of the graph involves rates and time periods far greater than any we would likely run into in our normal course of investment analysis, then my effort here will be just an academic exercise. On the otherhand, if the line is located such that it involves rates and time periods common in our day to day investment dealings, then we must be careful not to assume alternative rates of returns/time periods that are not supported by real world evidence. Evidence as to where the demarkation line may be located will be presented shortly. I will first address the shape of this demarkation line.

Figure 2

Figure 2 presents the three most likely shapes of the demarkation line between the feasible and infeasible regions. The actual line may contain segments of all three shapes, of course, but the overall shape will most likely be described by only one of them.

Shapes A and B can be quickly eliminated on the grounds that extending these lines to the right will cause them to cross the horizontal axis, thus making zero growth rates over extremely long periods of time "infeasible." Intuitively, we know this is an unacceptable conclusion.
Further evidence of a downward sloping line, concave from above (line C), is offered in figures 3 and 4. Figure 3 contains a rather typical growth curve..., one that is increasing at an increasing rate..., a growth curve that might easily be referred to as an exponential growth curve and be described by the formula: \( V_n = V_0 (1+i)^n \). However, in no way does this curve follow the growth path indicated by an exponential formula in which "i" retains a constant value. This can clearly be seen in figure 4, where selected constant-rate growth paths have been superimposed onto the original curve. At year-33 the curve has averaged a 10 percent compound growth rate (for earlier years the curve averaged significantly greater than 10 percent). By year-45 the average compound growth rate has fallen to 8 percent, and by year-72 to 6 percent. At year-120, where it intercepts the vertical axis, its average rate has fallen to 4.37 percent. Figure 5 presents the declining average compound growth rates of this growth curve plotted over selected time periods from 5 to 120 years. This new curve lends support to the conclusion regarding the shape of the demarkation line..., that it is downward sloping, concave from above.
Some evidence as to where (quantitatively) the demarkation line may be located.

Empirical evidence as to where this demarkation line is located must cover the full array of investment periods that we face in investment decisions; from a year or less through the 50 to 80-plus years involving investments in such things as power dams and timber growing. Value growth evidence for time periods of 5 to 10 years is not difficult to find. However, evidence covering time periods of 50 to 80-plus years is. Shortly, some time series evidence that has been found will be presented, but first the process of how these time series value data were analyzed will be briefly explained.

Suppose the value of something has been reported (recorded) every 10 years from 1930 through 1980. For each and every time period the compound rate of value increase (or decrease) can be calculated. In this example there are a total of 15 time periods; one 50-year period (1930-1980), two 40-year periods (1930-1970, 1940-1980), three 30-year periods, etc... and finally, five 10-year periods. Each of these 15 compound rates of value increase is then plotted over its respective time period. Actually, what is of interested is not all the plotted points, but only the maximum rate for each time period. These maximum rates would then be used to construct an envelope curve which in turn would be considered as one piece of evidence in the construction of the ultimate demarkation line, which would divide the feasible growth rate/time region from the infeasible region. Such a demarkation line would act as a limit to how long a given rate of value growth can reasonably be expected to continue..., a guide to reasonable and realistic discount rates (reasonable and realistic alternative rates of return).

Figure 6 contains the value growth rate/time plotted points of four historical time series. Figure 7 contains the approximate envelope curves for these four time series plottings. If the rather tight grouping of these four envelope curves reflects the grouping that is likely to occur when more time series data are analyzed, an overall envelope curve could easily be constructed which would be the sought for demarkation line. Furthermore, this demarkation line would be relevant to investment analysis in general because it does intervene into the rates of return and time periods often dealt with in investment evaluation.
Figure 6

Compound Rates of Value Increase of U.S. Farm Lands and Buildings Plotted Over Investment Duration (1850-1990)

Maximum Rates of Value Increase of Standard and Poor's "Industrials" (1871-1970)

Maximum and Minimum Compound Rates of Value Increase/Decrease of Real GDP Plotted Over Investment Duration (Dataset: 1850-1970) (Source: Bureau of Economic Analysis)


Figure 7

\[ r_i = (r - c) e^{-kt} + c \]
Furthermore, this formula allows for easy adjustments in the short-term, more current discount rates ($r_0$) that are needed periodically to reflect variations in the business and economic cycles. Figure 10 is an example of the extent the entire curve is affected by an $r_0$ adjustment of from 12% to a much higher 20%. The impact of such an adjustment is diminished significantly as the time period lengthens. Supposedly, once the long-term rate ($r_{\infty}$) is determined, it would seldom require further adjustment. Even then, it would only be of a very small magnitude.

![Figure 10](image)

$$\gamma = (\gamma - 2.5) e^{-k t} + 2.5$$

A possible modification of this formula-based rate determination approach would be a step approach (Figure 11), in which discount rates would decline, though be constant within specified investment time periods.

![Figure 11](image)

Advantages of Using this Approach for Selecting Discount Rates

In general, this discount rate selection approach would resolve many, if not most of the philosophical arguments that have developed over the years regarding appropriate discount rates...i.e., a high, easily adjustable rate to reflect current market conditions, versus a low, more stable rate that would recognize the claim that future generations have on economic resources. More specifically, this method would provide more realistic (evidence-supported) rates for more distant pay-offs; without being unrealistic for more immediate pay-offs, that is, provide current (higher) rates to evaluate short-term cash-flows and investments, and more realistic (lower) rates for evaluating distant cash-flows of long-term investments. In secondly, this method would allow short-term (current) rate adjustments to be made when needed without causing terminal "whip-lash" to long-term investments and investment proposals.
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