AN ABSTRACT

of

A Committee Report on

Definitions, Abbreviations, Information,
Mathematical Relationships and Formulas

Presenting Forestry Investment Analyses to Other Forestry
Specialists, Forest Landowners, and the General Public

Bennett B. Foster, Atlanta, GA, Chairman
Stephen M. Bratkovich, Jackson, OH
William C. Humphries, Macon, GA
Robert W. Mezger, Klamath Falls, OR
D. Lester Holley, Raleigh, NC
Allen L. Lundgren, New Brighton, MN
Jack Muench, Bethesda, MD

Purpose and Intent

Purpose: To suggest standard guidelines for analysts/authors who publish or otherwise present forestry investment related analyses in the more widely read professional and trade journals and magazines for audiences of other than economics and finance specialists and, directly or indirectly, for forest landowners/investors (decision-makers), and the general public.

Intent: To make such articles and reports more easily understood by non-specialists by using consistent terms, abbreviations and notations that have consistent meanings and interpretations; to present similar problems and analyses in such a way that their similarity is readily recognized; to allow those who accumulate files of investment oriented articles and reports over time to end up with a set of mutually comparable and supporting documents; and to assure that each article/report presents an adequate set of uninterpreted investment information so that readers, if they so desire, can reproduce the analysis findings and more widely compare the information with other published works.

These guidelines are not intended to be restrictive or exclusive. Other analytical formulas, terms, notations, abbreviations, etc. can and should be used when authors/analysts deem them more appropriate in particular cases or for particular audiences. However, unlike standardized formulas, terms, etc., non-standard terms, etc. must be accompanied by clear definitions and explanations in the text, footnotes or appendices. Whereas, standard terms, abbreviations, etc., once accepted, would not need to be constantly redefined.
**Standard Analytical Formulas**

1. **Reinstitution of the symbol 'p' in formulas**
   
e.g., \((1+p)^n\), where 'p' represents a generic "percentage" rate of average annual compound increase.

2. **Standard calculating formulas**

   Since most foresters have at least a passing familiarity with the formulas offered by K. P. Davis in his Forest Management text (Part 3), it is suggested that his formulas make up the standard set. (Appendix A)

3. **Defining the mathematical relationship among nominal, real, and inflation rates**

   i.e., \((1 + n) = (1 + r) (1 + i)\)
   where: \(n\), \(r\) and \(i\) are the decimal equivalents of the nominal, real and inflation rates, respectively.
   
   (If \(r = 8\%\) and \(i = 3.5\%\), then \(n = 11.78\%\) or \(11.8\%\))

   If authors feel that such technicalities or details are not appropriate for their intended audience, they should make the statement that the real rate plus the inflation rate is approximately equal to the nominal rate.

**Common measures of investment efficiency and their abbreviations**

1. **Internal Rate of Return (IRR)**

2. **Net Present Value (NPV) and Net Future Value (NFV)**

3. **Net Annualized Value (NAV)**

4. **Land Expectation Value (LEV)**

5. **Including the appropriate percentage rate when reporting NPVs, NAVs, LEVs, ...**

   e.g., \(NPV(X, \% ) = , or \ NPV_{X, \%} = \).²

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¹ Allegations of two "short-comings" of the internal rate of return have begun appearing in forestry investment literature during the last few years. The conclusion resulting from these allegations is that the IRR is not a valid, unambiguous measure of an investment's efficiency. Appendix B of this report contains arguments addressing these allegations.

² Even though the Benefit/Cost Ratio is not a term addressed in this report, attaching the appropriate rate to its abbreviation would be of similar value, i.e., \(B/C(X, \%) = , or B/C_{X, \%} = \).
**Standard information that should be included in all publications dealing with forest investment analyses.**

To make it possible for the reader to confirm the findings in published forestry investment analyses, it is suggested that a standard set of information be included in the text, in footnotes or in an appendix of all such published works. This standard information should include the following:

1. A precise statement of the magnitudes and timings of investment cash-flows, and whether they are in real or nominal terms. If real increases in future costs or revenues are assumed, include supporting evidence or a rationale for the assumed rates.

2. A precise statement of the discount and compound rate(s) used in the analysis, and whether it/they are in real or nominal terms. Include an explanation or rationale for selecting the rate(s) used.

**NOTE:** If cash-flows are in real terms then the discount/compound rate(s) should also be in real terms, and vice-versa.

3. If an analysis is in nominal terms, specify the inflation rate assumed and include an explanation or rationale for its selection.

4. If land values are explicitly considered investment costs, then they should also be assigned explicit end-of-investment values (revenues).

**Long-term and Short-term Inflation Rates, Real Rates and Discount Rates**

Don't be trapped into making long-term projections of short-term events.

Late '70s to early '80s ... high inflation, high investment rates, high mortgage rates ...

Many economists projected these high rates beyond the turn of the century.

- 200+ years of U.S. history ... average inflation: 1/2 - 3/4 %
- 40-60 year periods ... average: 1%, 4% max.
- 20-40 year periods ... average: 1%, 6% max.

Percentage gap between inflation and going nominal rates ...

- 1 1/2 - 2 1/2 points (low risk)
- 4 - 6 points (mod. risk)

Don't mix real rates with nominal rates!

This is a common error ...

cash-flow in real terms ..., discount rate in nominal terms
APPENDIX A
<table>
<thead>
<tr>
<th>Nature of Problem Solved</th>
<th>Formula 2/</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single payments</td>
<td></td>
</tr>
<tr>
<td>1. Future value of a single sum</td>
<td>( V_n = V_o (1+p)^n )</td>
</tr>
<tr>
<td>2. Present or discounted value of single sum</td>
<td>( V_o = V_n \left( \frac{1}{(1+p)^n} \right) )</td>
</tr>
<tr>
<td>2a. Value of ((1+p)), given (V_o), (V_n) and (n)</td>
<td>((1+p) = \left( \frac{V_n}{V_o} \right)^{1/n} )</td>
</tr>
<tr>
<td>2b. Value of (n), given (V_o), (V_n) and (p)</td>
<td>( n = \frac{\log V_n - \log V_o}{\log (1+p)} )</td>
</tr>
<tr>
<td>Multiple payments (term. equal ann. paymts.)</td>
<td></td>
</tr>
<tr>
<td>3. Future value of a series of (n) payments (future value reached the moment last payment is made)</td>
<td>( V_n = a \frac{(1+p)^n - 1}{p} )</td>
</tr>
<tr>
<td>4. Present or discounted value of a series of (n) paymts. (first payment made one year hence)</td>
<td>( V_o = a \frac{(1+p)^n - 1}{p(1+p)^n} )</td>
</tr>
<tr>
<td>5. Annual payment that will amount to a specified future sum (sinking fund payment) (future sum reached when last payment is made)</td>
<td>( a = V_n \frac{p}{(1+p)^n - 1} )</td>
</tr>
<tr>
<td>6. Annual payment that will pay off a specified current (capital) sum (installment payment) (first payment made one year hence, sum paid off when last payment is made)</td>
<td>( a = V_o \frac{p(1+p)^n}{(1+p)^n - 1} )</td>
</tr>
<tr>
<td>7. Future value of a series of (n) terminable periodic payments made (t) years apart (future value reached the moment last payment is made)</td>
<td>( V_n = a \frac{(1+p)^{nt} - 1}{(1+p)^t - 1} )</td>
</tr>
<tr>
<td>8. Present value of a series of (n) terminable periodic payments made (t) years apart (first payment made (t) years hence)</td>
<td>( V_o = a \frac{(1+p)^{nt} - 1}{<a href="1+p">(1+p)^t - 1</a>^{nt}} )</td>
</tr>
</tbody>
</table>
Multiple paymts. (infinite equal ann. payments)

9. Present or Capital value of a permanent annual income (first paymt. due one year hence)

\[ V_0 = \frac{a}{p} \]

10. Present or Capital value of a permanent periodic income (first paymt. due \( t \) years hence)

\[ V_0 = \frac{a}{(1+p)^t - 1} \]

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1/ Source: Forest Management (2nd Edition), K. P. Davis, Chapter 16, pp. 336-338

2/ Symbols used:

- **a** = amount of equal annual or periodic payment.
- **n** = number of years or interest-bearing periods.
- **p** = compound (or discount) rate per year or other specified interest-bearing period, expressed as a decimal.
- **t** = interval in years or number of interest-bearing periods between periodic payments, if other than one year.
- **\( V_0 \)** = value of a sum of money when placed at interest, or after it has been discounted to the present.
- **\( V_n \)** = value of a future sum of money, or the future value of an investment (after \( n \) years).
ALLEGED SHORTCOMINGS OF THE INTERNAL RATE OF RETURN AS A VALID MEASURE OF AN INVESTMENT'S PROFITABILITY

There are a number of examples in the literature in which the validity of the internal rate of return (IRR) as a measure of an investment's profitability has been challenged. These challenges are based on two alleged shortcomings:

1) The so-called "reinvestment assumption" regarding intermediate investment cash-flows, and

2) The possibility of multiple, therefore ambiguous, IRR's for investments with cash-flows containing more than one sign change.

This second so-called shortcoming has a related issue, which is that:

3) Multiple IRR's are much more likely if the cash-flow starts with a positive sign, a revenue.

1) Reinvestment Assumption

This alleged shortcoming is rooted in the traditional mathematical approach used to determine the IRR: Finding the discount rate that causes the NPV of an investment to equal zero. Implicit in this approach is that intermediate cash-flows (both positive and negative) are reinvested (or carried at a borrowing cost) at the sought-for IRR. If the approach is reversed, that is, by finding the compound rate that causes the Net Future Value (NFV) of an investment to equal zero, reinvestments (and carrying costs) at the sought for IRR are explicit. This is easily seen in the following examples:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash-flow</th>
<th>Present Value at 10% (discounted)</th>
<th>Future Value at 10% (compounded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-100</td>
<td>-100 -77 +38 +139</td>
<td>-100 +582 +158 -322 -418</td>
</tr>
<tr>
<td>Year 5</td>
<td>+61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 10</td>
<td>-200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 15</td>
<td>+582</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Since the NPV is zero (give or take some rounding error), the IRR = 10%. Also, since the NFV is zero, the IRR = 10%. Note that in the future value approach, carrying forward (compounding) the $61 explicitly assumes it was reinvested at the IRR (10%). It is also explicit that the "costs" (-$100 and -$200) were carried forward at a 10% borrowing rate.

The conclusion to this shortcoming argument is that, if an external reinvestment rate equal to the IRR is not available, then the IRR is not a valid measure of an investment's profitability. However, there is a mathematical approach for determining the IRR that does not assume any reinvestment or borrowing rate. The approach is simply to determine, through trial and error, the compound rate that precisely recreates the investment cash-flow.

Using the previous example, one can readily determine (verify) that 10% is the IRR. Think of the two negative cash-flows (-$100 and -$200) as deposits into some sort of account. Think of the two positive cash-flows ($61 and $582) as withdrawals from this account, the latter being the closeout withdrawal. The $100 beginning deposit is carried at 10% for five years, reaching a value of $161. Sixty-one dollars are withdrawn leaving $100. After 5 more years the value is again $161. An additional deposit of $200 results in a balance of $361. After 5 more years at 10% the value is $582. This $582 is withdrawn and the account is closed out. The cash-flow has been completely and precisely recreated. Note that how the withdrawals were used or where the deposits came from were not germane in determining the IRR. Such external factors are not relevant to the investment's IRR; the "reinvestment assumption" is not critical or even related to the validity of the IRR.

2) Multiple IRRs Due To Multiple Sign Changes In An Investment's Cash-flow.

This alleged shortcoming stems from "Descartes' Rule of Signs", a mathematical rule, which states that:

"The number of positive and negative roots (solutions) of a polynomial equation f(x)=0, with real coefficients, cannot exceed the number of variations in sign in f(x)...."

Just in case the significance of the words: "cannot exceed", escapes the reader, the authors of one elementary college algebra text 1/ include the following strongly worded caution:

"WARNING: Decartes' rule of signs merely gives the upper limit to the number of positive and negative roots of f(x)=0; there maybe fewer such roots, but never more."

Putting this rule into financial analysis terms: The number of IRR solutions of a complex investment cash-flow, where NPV=0, cannot exceed the number of sign changes in the investment cash-flow. Unfortunately, some analysts/authors have interpreted this rule as meaning that there will be as many IRR solutions as there are sign changes. They have confused a necessary condition for multiple IRR solutions with a sufficient condition.

It is important to understand that multiple sign changes is merely one of many necessary conditions that must exist in an investment cash-flow for multiple IRR solutions to exist. Another one is that the accumulative algebraic sum of the undiscounted cash-flow must suffer multiple sign changes also. Since this neces-

sary condition is much more definitive and restrictive, it could be presented as Decartes' modified rule of signs as it governs the number of IRR solutions of complex investment cash-flows:

The number of IRR solutions of a complex investment cash-flow, where NPV = 0, cannot exceed the number of sign changes in the accumulative algebraic sum of the undiscounted cash-flow.

WARNING: This rule merely gives the upper limit to the number of IRR solutions of NPV = 0; there may be fewer such solutions, but never more.

All three of the following investment cash-flows have multiple sign changes. However, only the third one has multiple sign changes in its accumulative algebraic sum. Furthermore, not even this third one has multiple IRR solutions.

<table>
<thead>
<tr>
<th>Year-0</th>
<th>Year-10</th>
<th>Year-20</th>
<th>Year-30</th>
<th>Year-40</th>
<th>Year-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>-100</td>
<td>+400</td>
<td>-250</td>
<td>+1</td>
<td>-50</td>
</tr>
<tr>
<td>2.</td>
<td>-150</td>
<td>-50</td>
<td>-50</td>
<td>+100</td>
<td>-50</td>
</tr>
<tr>
<td>3.</td>
<td>-180</td>
<td>+500</td>
<td>-500</td>
<td>+200</td>
<td>-25</td>
</tr>
</tbody>
</table>

The examples of multiple solution investment cash-flows that appear in the literature are usually contrived and bear little resemblance to any real life situations. This, along with the general lack of evidence to the contrary, supports the conclusion that the probability of actual investment cash-flows yielding multiple IRR solutions is so minute, that this alleged short-coming can be disregarded as a source of concern about the IRR's validity.

3) Cash-flows That Start With A Revenue

Cash-flows may start with either a positive sign or a negative sign. An example of one that starts with a positive sign, a revenue, is a budgetary cash-flow. Funds are received, followed by payments for goods or services purchased. Such a cash-flow might look like + - - - + - - -. However, it does not make sense to analyze this type of cash-flow for its IRR. Internal rates of return are meaningful only in the context of an investment, and an investment cash-flow is one that begins with a negative sign, an out-flow of funds.

When analysts fail to distinguish between 'budgeting-type' cash-flows and 'investment-type' cash-flows, the probability of multiple IRR solutions is greatly increased. One example of this occurring in forestry is when an existing stand of timber is to be harvested, followed shortly by regenerating for another stand, which will, in turn, be harvested at some later date. The cash-flow of this effort, + - + , will yield, according to Decartes' expanded rule of signs, either no IRR solution or two of them. This is not to be interpreted as support for the allegation that multiple IRR's do occur and should be a concern, but rather that improperly defined investment cash-flows will yield incorrect and meaningless IRR answers. Harvesting an existing stand is not part of the economic justification of regenerating the next stand. It is either economical to regenerate the next stand or it is not. From the stand point of economics, an existing stand's value should not in any way influence the economic justification of regenerating the stand after harvest. Policy may dictate that the existing stand will be harvested regardless of the economics of regeneration, or it may dictate that the existing stand will be harvested only if regeneration of the stand is economical. In either case, the value of the existing stand is not germane to the regeneration decision.