ESTIMATING UNCERTAIN TIMBER INVESTMENT
RETURNS FOR LANDOWNERS

DO RISK ADJUSTED DISCOUNT RATES
LEAD TO THE RIGHT DECISION?

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Abstract.--Non-industrial private landowners are faced with the problem of identifying the best use for their forest land. While their objectives in management may be broader than the monetary gains received, landowners still need reliable information on the types of economic returns available to them under different management options. However, estimating these returns is clouded by the uncertainty underlying the income flows produced. Usually, uncertainty is addressed by adding a risk factor to the discount rate used in present value calculations. However, finance theory suggests that an arbitrary adjustment for risk in the discount rate may not be consistent with the risk associated with a particular investment's cash flow. Furthermore, this problem is magnified when a single risk adjusted discount rate is used to evaluate many investment alternatives.

The purpose of this paper is to demonstrate how the use of risk adjusted discount rates may lead to improper investment ranking. By constructing various investment opportunities that differ in duration and risk, it is shown that relying on a risk adjusted discount rate does not steer individuals to the opportunities that maximize their welfare. The true preferred opportunity is identified by evaluating investments in terms of their certainty equivalents, a technique that adjusts each cash flow component separately for risk, rather than including a single adjustment in the discount rate. Furthermore, certainty equivalents directly incorporate how the investment's risk affects individual welfare. It is hoped that recognizing the deficiencies associated with risk adjusted discount rates will lead to improved investment guidelines for the landowner.

INTRODUCTION

One segment of forest economics research focuses on estimating the returns from stand level forest investments. These analyses, usually conducted on a single acre basis, estimate the returns to capital allocated to specific sites. As such, they provide information and decision guidelines to landowners who may be interested in managing timber as a source of income. However, underlying these analyses is the uncertainty in

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2 These types of analyses usually target management of a particular timber type in a given region. Examples include Kurtz et al. (1981), Kurtz et al. (1984) and Bailey (1986) for scarlet oak (Quercus coccinea Muenchh.), black walnut (Juglans nigra L.), and loblolly pine (Pinus taeda L.) in the southern
each management strategy's cash flow. The economic environment that determines the
future value of the timber is difficult to forecast, and the exact responses to silvicultural
traetnents, as well as harvest yields, are not known with certainty. Furthermore,
regeneration establishment is subject to losses, while mature stands are subject to threats
from wind, fire, and pests.

Ironically, the complex nature of forest investment uncertainty has led to analysis
techniques that rely on the tidiness found in deterministic models. One approach is to
rely on risk-adjusted discount rates (RADRs) in present value calculations. The idea
is simple. Best guess or average estimates are used to represent each investment's cash
flow and the choice of investment is based on a present value criterion such as net
present value (NPV) or internal rate of return (IRR). Adding a risk premium to the
discount rate (or cutoff rate of return in the IRR case) in effect penalizes the entire cash
flow for its risk, since the magnitude of future income generated is reduced in present
value under the higher rate. The larger the risk factor, the more risky the investment is
considered to be. Investment acceptance is more difficult, but those that survive the
higher rate are deemed worthy of the underlying risk.

The problem with the RADR method is that it is an indirect approach to risk in that
the source of risk lies in the uncertain cash flow, not the discount rate. In essence, using
a single a priori specified RADR may over or under compensate for the risk underlying
each cash flow component within an alternative, as well as, may over or under
compensate for the risk differences between alternatives (Lintner, 1965; Robichek and
Myers, 1966). As such, evaluating many alternatives with a single RADR interspecies biases in the ranking of alternatives for decision purposes. Furthermore, sensitivity
analyses around the discount rate do not alleviate the problem since little distinction is
made between the magnitude of the rate and the risk adjustment contained within.
Finally, in the context of an individual whose opportunity for diversification is limited,
knowing the appropriate RADR for each alternative involves solving the risk problem
via the certainty equivalent (CE) analogue to the RADR approach (Robichek and
Myers, 1966). Instead of adjusting the discount rate for risk, the CE approach adjusts
the expected (average) cash flow values for risk. Furthermore, the CE approach directly
incorporates how investment risk affects individual welfare.

Purpose

The purpose of this paper is to demonstrate the biases associated with risk adjusted
discount rates and how these biases may lead to improper investment ranking. The
analysis relies on the parallelism between the RADR and CE approaches to adjusting
for risk in investment evaluation. However, since the CE approach explicitly addresses
the risk found in the cash flow, it is considered superior to the RADR approach and

region respectively; and de Naurois and Buongiorno (1986) for red pine (Pinus resinosa Ait.) in the
northern lake states.

While there have been formal distinctions between risk and uncertainty, the terms are more or less
interchangeable. Uncertainty refers to future decision consequences not being known and risk connotes
a measurable reference to this notion.

For a review of the CE approach see Brigham (1985), pp. 436-443.

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therefore provides the correct evaluation of risk from which the biases associated with RADRs can be assessed (Robichek and Myers, 1966). After this parallelism is defined, the paper illustrates the problem of using a single \textit{a priori} specified RADR to evaluate a given investment alternative. This illustration is then extended to the case where many alternatives are being considered. The paper concludes by discussing the implications of relying on RADRs as an approach to risk, as well as, highlighting how sensitivity analyses can reduce the ambiguity underlying the choice of the discount rate (or rates).

**NOTATION**

This paper relies on the following notation. Let:

\begin{itemize}
  \item \( N \) - denote the last year of the investment horizon.
  \item \( CE_t \) - denote the certainty equivalent value for the uncertain cash flow component in year \( t \in \{0, 1, \ldots, N\} \) for a given investment opportunity (dollars).
  \item \( CF_t \) - denote the uncertain (i.e., random) cash flow component value in year \( t \in \{0, 1, \ldots, N\} \) for a given investment opportunity (dollars). Note: possible outcomes are represented by \( cf_t \).
  \item \( g \) - denote the risk adjusted discount rate appropriate for discounting the expected cash flow component value in year \( t \in \{0, 1, \ldots, N\} \) (percent/100).
  \item \( GD \) - denote the annual rate at which cash flow risk geometrically decreases over time (percent/100).
  \item \( GI \) - denote the annual rate at which cash flow risk geometrically increases over time (percent/100).
  \item \( k \) - denote the risk adjusted discount rate appropriate for discounting the entire expected cash flow for a given investment opportunity (percent/100).
  \item \( \text{LOG}(\cdot) \) - denote the natural logarithm function.
  \item \( MD \) - denote the annual rate at which cash flow risk linearly decreases over time (percent/100).
  \item \( MI \) - denote the annual rate at which cash flow risk linearly increases over time (percent/100).
  \item \( NPV \) - denote the uncertain net present value of the uncertain cash flow for a given investment opportunity (dollars). Note: the implied discount rate is the risk free rate.
  \item \( NPV_{CE} \) - denote the net present value of the certainty equivalent cash flow for a given investment opportunity (dollars). Note: the implied discount rate is the risk free rate.
  \item \( NPV_{RADR} \) - denote the net present value defined by discounting the expected cash flow for a given investment opportunity with a risk adjusted rate (or rates, dollars).
  \item \( r \) - denote the risk free rate of discount (percent/100, assumed constant over all future time periods).
\end{itemize}
\( \text{RP}_{\text{NPV}} \) - denote the net present value risk premium associated with holding a claim to a given investment opportunity (dollars).

\( \text{rp}_t \) - denote the risk premium in the discount rate for a given investment opportunity's uncertain cash flow component in year \( t \in \{0, 1, ..., N\} \) (percent/100).

\( \text{RP}_t \) - denote the cash flow component risk premium in year \( t \in \{0, 1, ..., N\} \) (dollars).

\( U(\cdot) \) - denote the individual's utility of wealth function.

\( \text{W}_0 \) - denote the individual's wealth endowment (dollars, assumed constant over all future time periods).

\( \text{W}_t \) - denote the uncertain future individual wealth position from holding claim to a given investment opportunity with an uncertain cash flow component in year \( t \in \{0, 1, ..., N\} \) (dollars). Note: possible outcomes are represented by \( \text{W}_t \).

\( \text{E}[\cdot] \) - denote the expectations operator. Note: \( \text{E}[\text{CF}_t] \) denotes the expected cash flow component value in year \( t \in \{0, 1, ..., N\} \) (dollars), \( \text{E}[U(\cdot)] \) denotes expected utility, and \( \text{E}[\text{NPV}] \) denotes expected net present value (dollars).

\( \alpha_t \) - denote the certainty equivalent adjustment factor applicable to a given investment opportunity's cash flow component in year \( t \in \{0, 1, ..., N\} \).

\( \sigma_{\text{CF}_t}^2 \) - denote the variance of \( \text{CF}_t \) (dollars squared).

\( \sim \) - be read as "indifferent to".

**ASSUMPTIONS**

The analysis focuses on a risk averse individual facing mutually exclusive investment opportunities that differ in duration (e.g., short versus long investment life), payoff (e.g., annual versus delayed), and cash flow risk (e.g., decreasing, constant, or increasing with respect to time). The individual's ability to diversify investments is assumed to be restricted, that is, the individual's portfolio will only contain the chosen investment opportunity. The individual's certain opportunity cost of invested capital will be defined by his or her marginal rate of time preference and represented by the risk free rate of discount (Hirshleifer, 1965). Furthermore, the analysis will be static in that the investment decision is being made today (i.e., year zero in the cash flow). Uncertainty is addressed by looking into the future and assessing how each uncertain cash flow component effects the individual's welfare at that time. For each investment alternative considered, the probability distribution for possible cash flow component outcomes will be normal and the variance of each cash flow component will define its risk.

\(^5\) For example, an individual landowner whose primary source of investment income is from managing his or her land for agricultural or timber (e.g., pulpwood or sawtimber) production.
Expected Utility and Risk Aversion

These assumptions allow the individual's welfare from adopting different investment opportunities to be measured under the mean-variance (E-V) form of the expected utility model. Underlying the expected utility model is the individual's utility function for wealth which in this analysis is assumed to be logarithmic and applicable in all future time periods. This function is defined as

\[ U(w_t) = \log(w_t) \]  

where:

\[ w_t = W_0 + c_f \]  \[ 1a \]

Logarithmic utility embodies the assumption of risk aversion in that marginal utility with respect to wealth is positive, but diminishing (i.e., the utility function is concave from below). That is, more wealth is better, but less so as you get more and more. Furthermore, logarithmic utility of wealth functions have the desired property that the individual's degree of risk aversion declines at higher wealth positions.  

THE PARALLELISM BETWEEN THE RADR AND CE APPROACHES TO RISK ADJUSTMENT

Consider some uncertain cash flow component in a given investment opportunity with some mean (expected) value, \( E[CF_t] \). The certainty equivalent CE, to the expected cash flow value \( E[CF_t] \) is defined as the amount that the individual is willing to receive with certainty such that the individual is indifferent between the certain amount and the expected amount with its associated risk. The exact value of the certainty equivalent will be determined by the expected value-risk tradeoffs implicit in the individual's expected utility function. This tradeoff is in turn defined by the probability distribution for the uncertain cash flow component. For logarithmic utility this relationship is given by

\[ U(W_0 + CE) = E[U(W_0 + CF)] \]  

where:

The expected utility model embodies the broadest category of risk assessment models in general and is based on a well defined set of preference ordering axioms. In a mean-variance application of expected utility, the axiomatic conditions imply that for a given expected (mean) outcome, the opportunity with the lowest risk (outcome variance) is the opportunity that gives the highest expected utility (i.e., maximizes welfare). For an excellent review of the expected utility model, see Anderson et al. (1977), pp. 65-100. Tobin (1958) demonstrates that the mean-variance case of the expected utility model holds when the probability distribution controlling uncertain outcomes is normal.

This property goes beyond the property of diminishing marginal utility with respect to wealth. The degree of risk aversion is measured by the coefficient of absolute risk aversion. Formally, the above property is referred to as decreasing absolute risk aversion with respect to wealth (see Anderson et al. (1977), pp. 89-90).

For a derivation of (2b) see Anderson et al. (1977), pp. 96-99. Equation (2b) assumes that the
\[ U(W_o + CE_t) = \log(W_o + CE_t) \quad [2a] \]

\[ E[U(W_o + CF_t)] = \log(W_o + E[CF_t]) - \frac{\sigma_{CF_t}^2}{2(W_o + E[CF_t])^2} \quad [2b] \]

That is, (2) specifies that the certainty equivalent value for the uncertain cash flow component is the value CE_t such that the utility from the certain wealth position \(W_o + CE_t\) (given by \(U(W_o + CE_t)\) in (2a)) equals the expected utility from holding claims to the uncertain cash flow component that defines the uncertain future wealth position \(W_o + CF_t\) (given by \(E[U(W_o + CF_t)]\) in (2b)). The relationship in (2b) explicitly defines the expected utility from holding claims to the uncertain cash flow component as a function of the mean outcome \(E[CF_t]\) and its variance \(\sigma_{CF_t}^2\).

The result for a risk averse individual is

\[ CE_t < E[CF_t] \quad [3] \]

since risk averse individual's are willing to accept lower expected cash flows for lower risk; and there is no risk in accepting the certain amount CE_t. Rearranging (3), the cash flow component risk premium is defined as

\[ E[CF_t] - CE_t = RP_t > 0 \quad [4] \]

The risk premium \(RP_t\) is the extra amount of expected cash flow value necessary to compensate the investor for the cash flow's underlying risk such that the investor is indifferent between the risky cash flow and the certain alternative.

In present value terms, the following then must also hold

\[ \left( \frac{CE_t}{(1 + r)^t} \right) \sim \left( \frac{E[CF_t]}{(1 + r)^t} \right) \quad [5] \]

since the individual is indifferent to receiving \(E[CF_t]\) with its risk and \(CE_t\) with certainty. Furthermore, since this relationship holds for all \(t \in \{0, 1, \ldots, N\}\), then

\[ NPV_{CE} \sim E[NPV] \quad [6] \]

where:

\[ NPV_{CE} = \sum_t \left[ \frac{CE_t}{(1 + r)^t} \right] \quad [6a] \]

\[ E[NPV] = \sum_t \left[ \frac{E[CF_t]}{(1 + r)^t} \right] \quad [6b] \]

Noting that the condition, \(CE_t < E[CF_t]\), holds for all \(t \in \{0, 1, \ldots, N\}\) in the risk averse case, then

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probability distribution for the cash flow component is normal (i.e., the higher moments beyond the distribution's variance are all zero).
\[ \text{NPV}_{CE} < E[NPV] \]  

The individual's risk premium (in present value terms) necessary to compensate the investor for the risk underlying the investment's entire cash flow is defined as

\[ \text{RP}_{NPV} = E[NPV] - \text{NPV}_{CE} > 0 \]  

As an alternative to the net present value risk premium in (8), the discount rate can be adjusted upward such that the risk premium in (8) vanishes. That is, the risk adjustment is now made in the discount rate such that

\[ \text{NPV}_{RA Dr} = \text{NPV}_{CE} \]  

Conceptually, there will be more than one RADR, one for each cash flow component in the investment alternative because for (9) to hold, the following must also hold

\[ \left( \frac{CE_t}{(1 + r)^t} \right) = \left( \frac{E[CF_t]}{(1 + g_t)^t} \right) \quad \text{for all } t \in \{0, 1, \ldots, N\} \]  

Furthermore, since \( CE_t < E[CF_t] \) holds for all \( t \in \{0, 1, \ldots, N\} \), then for (10) to hold, \( g_t > r \) and \( g_t \) can be structured as

\[ g_t = r + r p_t \]  

The proper discount rate adjustments to explicitly account for the individual's attitude toward the investment's cash flow risk (which is measured by the present value risk premium defined in (8)), are the discount rate risk premiums \( r p_t \) such that the equality in (10) holds for all \( t \in \{0, 1, \ldots, N\} \). Solving (10) for \( g_t \) and noting (11) results in

\[ g_t = \left[ \frac{E[CF_t]}{CE_t} \right]^t (1 + r) - 1 \]

or,

\[ r + rp_t = \left[ \frac{E[CF_t]}{CE_t} \right]^t (1 + r) - 1 \]

or,

\[ rp_t = \left[ \left( \frac{1}{\alpha_t} \right)^t - 1 \right] (1 + r) > 0 \]  

where:

\[ \alpha_t = \frac{CE_t}{E[CF_t]} \]  

[12a]

since for the risk averse case, \( 1/\alpha_t > 1 \). Note further that the ratio, \( 1/\alpha_t = E[CF_t]/CE_t \) is directly related to the cash flow component risk premium \( RP_t \) defined in (4). For a

Note that \( \text{NPV}_{RA Dr} \) in (9) should be interpreted as a different measure of the certainty equivalent \( \text{NPV}, \text{NPV}_{CE} \)
fixed expected cash flow component value $E[CF_i]$ the larger $R_{P_n}$, the larger the ratio $E[CF_i]/C_{E_n}$, and accordingly, the larger the risk adjusted discount rate $g_t$ or equivalently, the larger the discount rate risk premium, $r_{p_t}$. In fact, the inverse ratio $a_\tau$ (as defined in (12a)) is Robichok and Myers (1966) certainty equivalent adjustment factor and provides a scale free measure of the cash flow component's risk (the smaller $a_\tau$, the more risky the cash flow component is considered to be). Therefore, $a_\tau$ is an appropriate measure of the magnitude of the risk attitude of the investor toward the risky cash flow component.

Equation (12) explicitly shows that the appropriate risk adjustment in the discount rate is unique to each cash flow component. Two relations appear in (12). The appropriate risk premium $r_{p_n}$ is a function of

1) The investor's risk attitudes toward the uncertain cash flow component, $a_\tau$.

2) The timing of the cash flow component, $t$.

The implication in (12) is that a separate discount rate adjustment should be made for each cash flow component in the investment. That is, the net present value under the RADR approach should be calculated as follows

$$\text{NPV}_{\text{RADR}} = \sum_i \left[ \frac{E[CF_i]}{(1 + g_t)^i} \right] \quad [13]$$

Furthermore, there will be a single risk adjusted discount rate $k$ appropriate for discounting the entire expected cash flow such that

$$\text{NPV}_{\text{RADR}} = \sum_i \left[ \frac{E[CF_i]}{(1 + k)^i} \right] = \sum_i \left[ \frac{E[CF_i]}{(1 + g_t)^i} \right] \quad [14]$$

However, given a different investment alternative, with different risk underlying its cash flow, the solution to (14) will most likely result in a different single risk adjusted discount rate, $k$.

Therefore, underlying a correct application of the RADR approach are the following:

1) The choice of the appropriate discount rate adjustment is unique to each cash flow component in the investment alternative.

2) As a result of 1 above, there is no single a priori risk adjusted discount rate to use in evaluating all investment alternatives; or for that matter, to use in discounting all cash flow components in a single investment.

3) For a given investment alternative, the risk problem must already be solved (through the adjustment of the expected cash flow into its certainty equivalents) in order to determine the appropriate risk adjustment in the discount rate (or rates).

If a single a priori specified RADR is employed, one would expect investment alternatives with a lower implied risk to be biased toward unfavorable, while relatively higher risk investments would be biased as more favorable. Accordingly, the investment decision could lead to accepting alternatives that otherwise would be undesirable, or
rejecting alternatives that might otherwise be favorable, if their risk was explicitly assessed in the analysis. Herein lies the ambiguity of using an *a priori* specified risk adjusted discount rate.

ILLUSTRATING THE RADR BIAS

**Approach**

The approach taken to illustrate the RADR bias was to consider investment opportunities with the following risk trends underlying their cash flow: decreasing, constant, and increasing. Furthermore, under the increasing and decreasing trends, cash flow risk is assumed to increase or decrease at geometric and linear rates (table 1). For example, if risk is assumed to increase geometrically over time, this means that the variances in the cash flow components increase geometrically over time. If an opportunity has a single uncertain payoff 40 years in the future, this analysis assumes an implicit starting cash flow risk value in year 1 (the beginning of the future), say equal to 3,700,000 dollars squared. Then under the geometrically increasing trend (say at 3 %/year), the variance of the cash flow component in year 40 would be 11,717,799 dollars squared ( \( = 3,700,000(1.03)^{40-1} \)). Under the constant cash flow risk trend, the variance of the cash flow component in year 40 would be the 3,700,000 dollars squared starting value. Accordingly, if the cash flow risk trend is decreasing at a linear rate (say 2 %/year), the variance of the cash flow risk component in year 40 would be 814,000 dollars squared ( \( = 3,700,000(1 - 0.02(40 - 1)) \)).

The relationships in table 1 are used to construct hypothetical risk patterns underlying the uncertain cash flow for each investment alternative considered. The next step was to independently construct the certainty equivalent cash flow for each investment alternative such that all alternatives had the same certainty equivalent net present values. That is, the individual is indifferent to all alternatives considered. The final step then was to calculate the expected cash flow component values that were consistent with the certainty equivalent cash flow and the cash flow's risk values. This was accomplished by finding the solution for \( \mathbb{E}[CF_t] \) in (2b) (with \( W_o \) and \( \sigma_{CF_t}^2 \) in (2b) being known) such that the expected utility \( \mathbb{E}[U(W_o + CF_t)] \) equaled the certain utility from the certainty equivalent value, \( U(W_o + CE_t) \), (with \( CE_t \) in (2a) being known), thus having the relationship in (2) satisfied. For example, if \( W_o = \$20,000.00, \sigma_{CF_t}^2 = 1,397,034 \) dollars squared, \( CE_t = 1,889.96 \) dollars, then the expected cash flow component solution value for year \( t \) of the cash flow would be 1,921.82 dollars since in (2b)

\[
\mathbb{E}[U(W_o + CF_t)] = \text{LOG}(21921.82) - \frac{1397034}{2(21921.82)^2} = 9.993784
\]

which equals

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Table 1. Cash flow risk trends used in the analysis.

<table>
<thead>
<tr>
<th>I) Decreasing cash flow risk with respect to time$^{(1)}$</th>
<th>II) Constant cash flow risk with respect to time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear trend: $\sigma^2_{\tilde{X}<em>t} = \sigma^2</em>{\bar{X}}(1 - MD(t - 1))$</td>
<td>Linear trend: $\sigma^2_{\tilde{X}<em>t} = \sigma^2</em>{\bar{X}}$</td>
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<tr>
<td>Geometric trend: $\sigma^2_{\tilde{X}<em>t} = \sigma^2</em>{\bar{X}}(1 - GD)^{t-1}$</td>
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<tr>
<td>III) Increasing cash flow risk with respect to time</td>
<td></td>
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<tr>
<td>Linear trend: $\sigma^2_{\tilde{X}<em>t} = \sigma^2</em>{\bar{X}}(1 + MI(t - 1))$</td>
<td>Geometric trend: $\sigma^2_{\tilde{X}<em>t} = \sigma^2</em>{\bar{X}}(1 + GI)^{t-1}$</td>
</tr>
</tbody>
</table>

Notes:
$^{(1)}$ When risk is decreasing under the linear trend, cash flow component risk can conceivably go to zero and remain at zero thereafter.

$\sigma^2_{\tilde{X}_t}$ - variance of the uncertain cash flow component in year $t$ (dollars squared).

$\sigma^2_{\bar{X}}$ - starting cash flow variance value (implied cash flow component risk in year 1 of the cash flow) (dollars squared).

MD - annual rate of linear decrease (%/100).

GD - annual rate of geometric decrease (%/100).

MI - annual rate of linear increase (%/100).

GI - annual rate of geometric increase (%/100).

\[
U(W_o + CE_t) = \log(21889.96) = 9.993783
\]
in (2a).$^{10}$

This approach is backwards in that in an actual situation $E[CF_t]$ and $\sigma^2_{\tilde{X}_t}$ would be known (as well as the utility of wealth function parameter, $W_o$), and the relationship in (2) would be solved for the certainty equivalent value, $CE_t$. However, since this analysis is designed to show the biases from using a priori specified RADRs, the backwards approach allows for the set up of true indifference between alternatives (by equating their certainty equivalent NPVs) and then solving for the implied expected cash flow values that, given the cash flow risk values, would yield the predetermined certainty equivalent cash flow. In this manner, explicit ranking of the investment alternatives

$^{10}$ The relationship in (2) was solved iteratively, with convergence defined by taking the antilog of (2a) and (2b) and having them be within 0.001 antilog units of each other.
under a chosen RADR would reflect biases in the ranking since the correct ranking is one of indifference.

**RAR Bias within the Evaluation of an Investment Alternative**

To illustrate this approach, consider an investment opportunity with a certain cost (in year zero) of $1,000.00 and uncertain payoffs in years \( t = 10, 20, \ldots, 60 \). Suppose that the risk free discount rate is 2 percent \( (r = 0.02) \) and the certainty equivalent NPV is $5,000.00 \( (\text{NPV}_{\text{CE}} = 5000.00) \). Then a certainty equivalent cash flow of \( \text{CE}_t = 1889.96 \) dollars for \( t = 10, 20, \ldots, 60 \) would yield a present value of the certainty equivalent payoffs of

\[
\frac{1889.96((1.02)^{60} - 1)}{((1.02)^{10} - 1)(1.02)^{50}} = 6000
\]

such that the target value, \( \text{NPV}_{\text{CE}} = 5000.00 \) dollars is met. Table 2 shows the expected cash flow values under different cash flow risk assumptions when the individual's wealth endowment is $20,000.00 \( (W_o = 20000.00) \). Table 3 gives similar values for the cash flow for the case where the individual's wealth endowment is $60,000.00 \( (W_o = 60000.00) \).

Shown in tables 2 and 3 are the certainty equivalent cash flow values, assumed to be constant under all cash flow risk trends and the wealth endowment assumption (i.e., regardless of the cash flow risk trend and endowment assumption, the certainty equivalent NPV is $5,000.00). Therefore, the difference between tables 2 and 3 is in the required expected cash flow component values necessary to compensate for the cash flow component's risk, such that the individual is indifferent between the three alternatives shown (defined by the cash flow risk trend). Due to decreasing absolute risk aversion with respect to wealth, the wealthier individual \( (W_o = 60000.00, \text{table 3}) \) requires a lower expected cash flow (i.e., lower cash flow component risk premiums) that the less endowed individual \( (W_o = 20000.00, \text{table 2}) \). This is confirmed by comparing the certainty equivalent adjustment factors for the two individuals (the \( \alpha \) in (12a), the CE ADJ FAC values in table 2 and 3), the higher factors for the wealthier individual (table 3) implying a lower attitude toward cash flow risk. However for both individuals, the larger the cash flow component's risk, the larger the expected cash flow component value must be to compensate for this risk.

Sidestepping from the backwards approach in setting up the cash flows, tables 2 and 3 can also be viewed as representing three investment opportunities that each individual faces. For the less endowed individual (table 2), the cash flow components' expected values and variances\(^{11}\) define relatively lower risk opportunities than the opportunities faced by the wealthier individual (table 3). This can been seen by comparing the probability that each cash flow component will actually be a loss (the PLOSS values in tables 2 and 3). That is, the cash flow component probability distributions faced by the wealthier individual have means (expected values) and variances (risk values) that define a higher probability of loss for each cash flow component, when compared to the means

\(^{11}\) The standard deviations in tables 2 and 3 are included to give a better feel for the magnitude of the cash flow variances.
Table 2. Example of cash flows for a given investment type under differing cash flow risk trends.

WEALTH ENDOWMENT = $20,000.00  
RISK FREE RATE OF DISCOUNT = 0.02

<table>
<thead>
<tr>
<th>YEAR</th>
<th>EXP CF</th>
<th>RISK</th>
<th>CE CF</th>
<th>CE ADJ FAC</th>
<th>ST DEV</th>
<th>PLOSS</th>
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<td></td>
<td>Decreasing Cash Flow Risk: Linear Trend MD = 0.02</td>
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<tr>
<td>10</td>
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<td>5207121</td>
<td>1889.96</td>
<td>0.9412</td>
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<tr>
<td>20</td>
<td>1979.33</td>
<td>3937092</td>
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<td>0.9548</td>
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<tr>
<td>40</td>
<td>1921.82</td>
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<td>1889.96</td>
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<td>0.00</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

|      | Increasing Cash Flow Risk: Geometric Trend GI = 0.03 | | | | | |
| 10   | 2076.82 | 8285440 | 1889.96 | 0.9100 | 2878.44 | 0.2353 |
| 20   | 2139.99 | 11134841| 1889.96 | 0.8832 | 3336.89 | 0.2607 |
| 30   | 2224.10 | 14964183| 1889.96 | 0.8498 | 3868.36 | 0.2827 |
| 40   | 2335.64 | 20110416| 1889.96 | 0.8092 | 4484.46 | 0.3012 |
| 50   | 2483.04 | 27026496| 1889.96 | 0.7611 | 5198.70 | 0.3165 |
| 60   | 2576.83 | 36321024| 1889.96 | 0.7060 | 6026.69 | 0.3285 |

|      | Increasing Cash Flow Risk: Geometric Trend GI = 0.06 | | | | | |
| 10   | 2131.02 | 10728346| 1889.96 | 0.8869 | 3275.42 | 0.2576 |
| 20   | 2316.31 | 19212640| 1889.96 | 0.8159 | 4383.22 | 0.2986 |
| 30   | 2537.31 | 34406576| 1889.96 | 0.7166 | 5865.71 | 0.3265 |
| 40   | 3181.59 | 61616368| 1889.96 | 0.5940 | 7849.61 | 0.3426 |
| 50   | 4075.87 | 110344336| 1889.96 | 0.4637 | 10504.49| 0.3490 |
| 60   | 5486.27 | 197608080| 1889.96 | 0.3445 | 14057.31| 0.3482 |

Notes:

YEAR - year in which the uncertain cash flow component occurs.
EXP CF - expected cash flow component value (dollars).
CE CF - certainty equivalent cash flow component value (dollars).
RISK - variance of the uncertain cash flow component (dollars squared).
CE ADJ FAC - certainty equivalent adjustment factor.
ST DEV - standard deviation of the uncertain cash flow component value (dollars).
PLOSS - probability that the cash flow component value outcome is less than zero.
MD - annual rate of linear decrease in cash flow risk (%/100).
GI - annual rate of geometric increase in cash flow risk (%/100).
Table 3. Example of cash flows for a given investment type under differing cash flow risk trends.

**WEALTH ENDOWMENT = $60,000.00**

**RISK FREE RATE OF DISCOUNT = 0.02**

<table>
<thead>
<tr>
<th>YEAR</th>
<th>EXP CF</th>
<th>RISK</th>
<th>CE CF</th>
<th>CE ADJ FAC</th>
<th>ST DEV</th>
<th>PLOSS</th>
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<tr>
<td></td>
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**Decreasing Cash Flow Risk: Linear Trend**  **MD = 0.02**

<table>
<thead>
<tr>
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<th>RISK</th>
<th>CE CF</th>
<th>CE ADJ FAC</th>
<th>ST DEV</th>
<th>PLOSS</th>
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<td>6026.69</td>
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</table>

**Increasing Cash Flow Risk: Geometric Trend**  **GI = 0.03**

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<tr>
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<th>EXP CF</th>
<th>RISK</th>
<th>CE CF</th>
<th>CE ADJ FAC</th>
<th>ST DEV</th>
<th>PLOSS</th>
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<tr>
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<td>0.3963</td>
</tr>
<tr>
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</tr>
</tbody>
</table>

**Increasing Cash Flow Risk: Geometric Trend**  **GI = 0.06**

Notes:

- **YEAR** - year in which the uncertain cash flow component occurs.
- **EXP CF** - expected cash flow component value (dollars).
- **CE CF** - certainty equivalent cash flow component value (dollars).
- **RISK** - variance of the uncertain cash flow component (dollars squared).
- **CE ADJ FAC** - certainty equivalent adjustment factor.
- **ST DEV** - standard deviation of the uncertain cash flow component value (dollars).
- **PLOSS** - probability that the cash flow component value outcome is less than zero.
- **MD** - annual rate of linear decrease in cash flow risk (%/100).
- **GI** - annual rate of geometric increase in cash flow risk (%/100).
and variances of the probability distributions faced by the less endowed individual (this difference being attributed to the differences in the respective means). Therefore, when the investment opportunities are viewed in the forward direction (by taking the cash flow component mean and variance values as given), the wealthier individual faces a riskier opportunity set. However, both individuals are indifferent between any alternative in their respective opportunity sets because the wealthier individual makes a relatively lower risk adjustment than the less endowed individual (as indicated by their certainty equivalent adjustment factors), again because the wealthier individual has a lower degree of risk aversion.

This comparison illustrates how different individuals can have different attitudes regarding investment risk. Table 4 gives the implied correct risk adjusted discount rates for the two individuals such that the true indifference between the 3 opportunities is met for both individuals. Table 4 confirms the relationship in (12) that the correct risk premium in the discount rate$^{12}$ is unique to the cash flow component being considered.$^{13}$ Furthermore, table 4 shows that the correct risk premium is also unique to the individual. Again due to decreasing absolute risk aversion with respect to wealth, the wealthier individual ($W_0 = \$60000.00$) has lower implied RADRs than the less endowed individual ($W_0 = \$20000.00$). Table 4 also gives the correct single RADR to discount each investments opportunity's entire expected cash flow with (the $k$ in (14), and the INV RADR in table 4). Intuitively, these rates should strike a middle ground between the values for the implied cash flow component rates. This is confirmed in table 4. Again, the implied single risk adjusted rate is unique to the riskiness of the investment opportunity, as well as, the individual.

Also note that the implied cash flow component RADRs do not necessarily increase as the cash flow component risk increases (as indicated by the lower values for the certainty equivalent adjustment factors). Only when cash flow component risk blows up (e.g., increases geometrically at 6% per year), do the implied cash flow component RADRs increase. This is more dramatic in the case of the less endowed individual ($W_0 = \$20000.00$). However, the implied single RADRs appropriate for discounting the entire expected cash flow tend to reflect the overall riskiness of the investment. For example, in the case of the less endowed individual ($W_0 = \$20000.00$) the decreasing cash flow risk case can be considered as being a low risk investment and has an implied single RADR of 0.0212. Accordingly, the increasing risk case (geometric trend, GI = 0.06) can be considered as a high risk investment, and therefore has a higher implied single RADR equal to 0.0341 ($W_0 = \$20000.00$).$^{14}$

---

$^{12}$ Given by taking the cash flow component RADRs (the $g_i$ in (10), (11), and (13); and the CF RADRs in table 4) and subtracting off the 2 percent risk free component.

$^{13}$ With the exception of the cash flow component in year 10, for the alternative with geometrically increasing cash flow risk (GI = 0.03 in table 4), a constant RADR is implied. This will be the case when the certainty equivalent adjustment factor is decreasing at a constant rate defined by the constant risk premium in the discount rate (Robichek and Myers, 1966).

$^{14}$ Note that the risk premium in this case (0.0141) represents 41 percent of the risk adjusted discount rate's magnitude, or a risk adjusted rate 71 percent greater than the 2 percent risk free rate.
### Table 4. Example of correct risk adjusted discount rates to use in investment evaluation.

**RISK FREE RATE OF DISCOUNT = 0.02**

\[
\begin{align*}
W_e &= $20000.00 \\
W_e &= $60000.00
\end{align*}
\]

<table>
<thead>
<tr>
<th>YEAR</th>
<th>CE ADJ FAC</th>
<th>CF RADR</th>
<th>CE ADJ FAC</th>
<th>CF RADR</th>
</tr>
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<tbody>
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</tbody>
</table>

**INV RADR = 0.0212**

**Decreasing Cash Flow Risk: Linear Trend**  MD = 0.02

<table>
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<th>YEAR</th>
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<th>CF RADR</th>
<th>CE ADJ FAC</th>
<th>CF RADR</th>
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**INV RADR = 0.0261**

**Increasing Cash Flow Risk: Geometric Trend**  GI = 0.03

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<th>CF RADR</th>
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</thead>
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<td>0.0334</td>
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<tr>
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<td>0.4637</td>
<td>0.0358</td>
<td>0.6840</td>
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<td>60</td>
<td>0.3445</td>
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<td>0.5512</td>
<td>0.0302</td>
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</table>

**INV RADR = 0.0341**

**Increasing Cash Flow Risk: Geometric Trend**  GI = 0.06

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<th>CF RADR</th>
<th>CE ADJ FAC</th>
<th>CF RADR</th>
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<td>0.3445</td>
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</table>

**INV RADR = 0.0266**

**Notes:**
- **W_e**: individual's wealth endowment.
- **YEAR**: year in which the uncertain cash flow component occurs.
- **CE ADJ FAC**: certainty equivalent adjustment factor.
- **CF RADR**: implied correct risk adjusted rate to discount the expected cash flow component value with (%/100).
- **INV RADR**: implied correct risk adjusted rate to discount the entire expected cash flow with (%/100).
- **MD**: annual rate of linear decrease in cash flow risk (%/100).
- **GI**: annual rate of geometric increase in cash flow risk (%/100).
RADR Bias in the Evaluation of Many Investment Alternatives

The preceding illustration highlights that as long as investment alternatives differ in cash flow risk, there is no single correct RADR to evaluate all investment alternatives with. This section illustrates the biases associated with using a single a priori RADR by considering three investment types (figure 1). For each type, there are alternatives defined by one of the cash flow risk trends in table 1, for a total of 15 alternatives considered.\textsuperscript{15} The sawtimber investment represents an investment opportunity with a single initial cost and a delayed uncertain payoff in year 40. The pulpwood investment represents the same type of investment but under a shorter duration (20 years). The crop investment represents an even shorter duration investment (10 years) with annual (beginning in year 1) uncertain payoffs. As indicated in figure 1, the short duration investments are repeated such that all investment alternatives have a common life. Furthermore, the entire investment horizon (N = 40) is considered to be uncertain, that is, when the shorter duration investments are repeated, the risk trends defined in table 1 are still in effect.

In this analysis, the risk free rate of discount was 2 percent. For simplicity, investment costs were considered certain and each investment had the same $1,000.00 capital requirement (in present value dollars, future costs being discounted with the risk free rate). Furthermore, all investment alternatives (defined by a particular investment type, cash flow risk trend combination) were constructed to have a certainty equivalent NPV of $5,000.00. This was accomplished by defining a constant certainty equivalent cash flow for each investment type that met a present value target of $6,000.00 when discounted at the 2 percent risk free rate.\textsuperscript{16} Using the relationship in (2), the expected values for the uncertain payoffs were determined such that they were consistent with the known certainty equivalent and risk values for each of the cash flow components in the 15 investment alternatives considered.\textsuperscript{17}

The next step was to evaluate the 15 alternatives (i.e., discount their expected cash flows) under various RADRs, ranging from 2.5 to 6.0 percent (that is, containing risk premiums ranging from .5 to 4.0 percent). However, since costs were assumed to be certain, they were still discounted with the risk free rate.\textsuperscript{18} Figure 2 diagrams the results

\textsuperscript{15} As was the case in the previous section, only a sample of the results will be presented.

\textsuperscript{16} For the sawtimber investment, this gave a single certainty equivalent payoff in year 40 equal to $13,247.80. For the pulpwood investment, this gave two certainty equivalent payoffs in years 20 and 40 equal to $5,329.13. For the crop investment, this gave a annual certainty equivalent payoff (beginning in year 1) equal to $219.34 per year.

\textsuperscript{17} In this analysis, the starting risk values used to generate the cash flow variances were scaled such that they were consistent with the magnitude of the expected cash flow. This was accomplished by selecting a starting risk value such that the probability of incurring a loss was around 0.16. An initial expected cash flow starting value (used only for scaling the starting risk value) was estimated by assuming a certainty equivalent adjustment factor of 0.75.

\textsuperscript{18} Note that an added bias would have been introduced if the certain costs in the pulpwood and crop
Figure 1. Three investment types that differ in duration and payoff pattern.

C - denotes certain investment cost (dollars).
R - denotes uncertain payoff (dollars).
\(/\) indicates that the investment is repeated.
\(\ldots\) indicates an annual payoff flow.
Figure 2. Net present value results for the evaluation of 3 investment types using risk adjusted discount rates.

SAW(I) - sawtimber investment with geometrically increasing cash flow risk (rate of increase is 6%/year).

PULP(I) - pulpwood investment with geometrically increasing cash flow risk (rate of increase is 6%/year).

CROP(I) - crop investment with geometrically increasing cash flow risk (rate of increase is 6%/year).

PULP(C) - pulpwood investment with constant cash flow risk.

SAW(D) - sawtimber investment with geometrically decreasing cash flow risk (rate of decrease is 2%/year).
of the analysis for the case where the wealth endowment to the individual was $20,000.00, while figure 3 diagrams the results of the analysis for the case where the wealth endowment was $60,000.00.\footnote{19}

Indicated in figures 2 and 3 is the certainty equivalent NPV of $5,000.00. The curves in figures 2 and 3 represent the corresponding NPV estimates under the various risk adjusted discount rates. The intersection of these curves with the certainty equivalent NPV line gives the correct single RADR (the k in (14)) to discount the entire cash flow with. For example, in figure 2 the sawtimber investment with geometrically increasing cash flow risk (GI = 0.06, SAW(I) curve in figure 2) is 4.5 percent. Accordingly, the correct single RADR for the crop investment with geometrically increasing cash flow risk (GI = 0.06, CROP(I) curve in figure 2) is 2.18 percent. Comparison of these rates implies that while risk is increasing geometrically for both alternatives, the crop investment is less risky because the annual payoffs in effect dilute the risk. However, in the sawtimber investment, the risk is concentrated in the single uncertain cash flow payoff in year 40.

The amount of deviation of the NPV curves from the certainty equivalent NPV line reflects the bias from using an incorrectly specified RADR to evaluate all of the investment alternatives with. For every rate considered, some alternatives are incorrectly valued. For example, if the chosen RADR is 4 percent in figure 2, all the investment alternatives with an implied correct RADR less than 4 percent (the CROP(I), PULP(C), and SAW(D) alternatives) are under valued, reflecting too high an adjustment for risk. However, the SAW(I) and PULP(I) alternatives (with implied correct RADRs greater than 4 percent) are over valued, reflecting too low an adjustment for risk. The result is that the SAW(I) alternative would be incorrectly ranked as the preferred alternative.

The slope of the NPV curves in figures 2 and 3 indicate how sensitive the investment's valuation is to the magnitude of the risk adjustment in the discount rate. For example, the sawtimber and pulpwood investments (which have delayed payoffs) had been discounted with risk adjusted rates. Under the higher rates, the present value of the future costs would be reduced in magnitude, thus favoring these investments relative to the sawtimber investment. This provides a simple example of how over compensating for risk (in this case on the cost side) biases the analysis.

\footnote{19} Figures 2 and 3 diagram the results for 5 of the 15 investment alternatives considered. They are

- **SAW(I)** - sawtimber investment with geometrically increasing cash flow risk (rate of increase is 6% per year).
- **PULP(I)** - pulpwood investment with geometrically increasing cash flow risk (rate of increase is 6% per year).
- **CROP(I)** - crop investment with geometrically increasing cash flow risk (rate of increase is 6% per year).
- **PULP(C)** - pulpwood investment with constant cash flow risk.
- **SAW(D)** - sawtimber investment with geometrically decreasing cash flow risk (rate of decrease is 2% per year).

The alternatives were chosen because they highlight the biases encountered in all 15 alternatives considered. Note that figures 2 and 3 are similar, indicating that the biases incurred are insensitive to the wealth endowment assumption.
WEALTH ENDOWMENT = $60000.00

Figure 3. Net present value results for the evaluation of 3 investment types using risk adjusted discount rates.

SAW(I) - sawtimber investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).

PULP(I) - pulpwood investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).

CROP(I) - crop investment with geometrically increasing cash flow risk (rate of increase is 6 %/year).

PULP(C) - pulpwood investment with constant cash flow risk.

SAW(D) - sawtimber investment with geometrically decreasing cash flow risk (rate of decrease is 2 %/year).
are more sensitive to an incorrect specification of the RADR than the crop investment (which has an annual payoff). This is consistent with traditional sensitivity analyses with respect to the magnitude of the discount rate; low rates tend to favor delayed payoff investments while higher discount rates tend to have a harsher effect on these types of investments. The result is that under different RADRs, there are differences in the ranking of the alternatives (albeit, an incorrect ranking, recall that the true ranking is one of indifference). For example, in figure 3, under the 2.5 percent RADR, the CROP(I) alternative is ranked relatively low, while the PULP(I) and SAW(I) investments are ranked relatively high. However, when the RADR is chosen to be 6 percent in figure 3, the CROP(I) investment outranks all alternatives. Therefore, a single a priori specified RADR will not only bias the valuation of each investment alternative, but the relative desirability between alternatives as well.

**DISCUSSION**

The following generalizations can be made from the preceding illustrations.

1) Risk adjustments in the discount rate should reflect the individual's perceptions regarding the riskiness of cash flow components. This implies an unique risk adjustment in the discount rate for each cash flow component in the alternative.

2) Increasing cash flow risk with respect to time does not necessarily imply using higher rates to discount more distant cash flow components.

3) If a single RADR is chosen to discount the alternative's entire expected cash flow, the risk adjustment should reflect the individual's perceptions regarding the overall risk underlying the investment. This implies an unique single adjustment in the discount rate for each investment alternative considered.

4) Using a single RADR in the evaluation of many investment alternatives in effect ignores the relative riskiness between the alternatives considered.
   a) Applying a RADR that is too low tends to over value relatively high risk investments due to an under compensation for risk.
   b) Applying a RADR that is too high tends to under value relatively low risk investments due to an over compensation for risk.

5) Over and under valuation biases tend to be greater in the evaluation of investments that have intermittent delayed payoffs, even for small errors in the risk adjustment.

6) If the chosen RADR is too high, the tendency is to favor short duration, annual payoff investments.

All these generalizations point to the fact that when investments are evaluated with risk adjusted discount rates, there is rarely ever a single correct RADR applicable to all the alternatives considered. The implication in forestry is that even when investment alternatives differ only in terms of a single parameter (say the rotation age), there will most likely be an implied RADR unique to each alternative. Furthermore, for even-age management strategies, where the cash flow contains intermittent delayed payoffs, small errors in the discount rate will have large impacts on the over or under valuation of the investment thus making it difficult to correctly rank these alternatives against the short
duration, annual payoff investments, typical of agricultural alternatives. The paradox is, knowing the appropriate RADRs to apply to each alternative necessitates having already solved the investment evaluation problem through the use of certainty equivalents.

While this paper focused on the individual, the generalizations in 1 through 4 above will hold in the case of corporate sector analysis (Lintner, 1965; Fama, 1977). The only modifications are that investment risk is defined in terms of its systematic risk (i.e., non-diversifiable market risk) and the certainty equivalents to the expected cash flow are market determined (Lintner, 1965). However, if the risk free rate is assumed to be constant over time, and the project investments considered do not change the firm’s systematic risk (i.e., consistent with the firm’s line of business), then using a single RADR applicable to all project investments can be justified (Fama, 1977). But in as much as project acceptance changes the firm’s systematic risk, each alternative will have a uniquely defined appropriate RADR to use in the discounting of the expected cash flow. This ambiguity may explain why only 58 percent of surveyed forest products firms rely on the RADR approach to account for risk in project evaluation; the other 40 percent mainly rely on subjective adjustments to the cash flow, as well as, focussing on the variability in the investment’s returns (i.e., principles underlying the CE approach to risk) (Redmond and Cubbage, 1985).

Implications Regarding Sensitivity Analysis around the discount rate

Traditional sensitivity analyses around the discount rate usually focus on how sensitive the investment evaluation is to the magnitude of the rate. However, little attention is paid toward whether differences in the size of the discount rate reflect differences in the size of the risk free component, the risk premium, or in both components. For that matter, little recognition is given to whether the discount rate contains an explicit adjustment for risk. Furthermore, by focusing on the magnitude of the entire rate, these analyses tend to reinforce the notion that a single appropriate rate must ultimately be chosen and the investment decision be made accordingly. However, as the results of this analysis indicate, nothing could be farther off the mark.

Given that the fundamental problem with using RADRs is not knowing what rate is appropriate for each alternative, sensitivity analyses around the discount rate should be presented such that the risk free and risk premium components of the discount rate are separated out. For each investment alternative considered, a NPV matrix can be defined, where the rows represent possible values for the risk free rate and columns represent possible values for the risk premium (figure 4). For example, suppose the sensitivity analysis was around discount rates of 2, 3, 4, and 5 percent and the NPV_{RADR} of the investment (calculated by discounting the expected cash flow) are $4,000.00, $2000.00, $500.00, and -$500.00 respectively. As shown in figure 4, a 4 percent discount rate could imply a risk free rate of 4 percent; or risk adjusted rates with

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20 For public investment evaluation, there is the more fundamental question as to whether the government should even consider risk in investment evaluation. See Hirschleifer (1964), Samuelson (1964), Vickrey (1964), and Hirschleifer (1966) for the foundations of this issue.

21 See Brigham (1985), pp. 395-416; especially pp. 414-415, for a practical approach to this problem.
Figure 4. Example of a sensitivity analyses layout that separates out the risk free and risk premium components of the discount rate.

Values in the matrix correspond to the net present value (dollars) under a given assumption regarding the discount rate's risk free and risk premium component values. Adding these two components gives the magnitude of the discount rate.

risk premiums of 1 and 2 percent, with corresponding risk free components of 3 and 2 percent.

The advantage to the layout in figure 4 is that each element in the matrix gives the NPV valuation for a specific assumption regarding the risk free rate, as well as the adjustment for risk. Therefore, the decision maker need only worry about the appropriate choice for the risk free component, and then for each alternative, rely on subjective perceptions about the relative riskiness of the alternatives. In this manner, the choice for each alternatives risk premium can be made and the alternatives evaluated accordingly. Such a presentation provides more complete information about
the investment analysis; namely it facilitates choosing a unique RADR applicable to each alternative considered.\footnote{22}

**Further Research**

The CE framework provides the theoretical analogue from which the appropriateness of the RADR approach can be questioned. However, in actual application, the CE approach has a fundamental weakness; namely that the utility function for wealth must be known for all relevant future time periods. In the case of logarithmic utility, this is tantamount to knowing the future wealth position of the individual (i.e., $W_0$) in all future time periods. But if this information is known, then surely, the future cash flow values must be known as well.

Furthermore, this analysis was constructed to dramatize the pitfalls associated with using risk adjusted discount rates. Empirical research needs to be developed to estimate the risk trends underlying forest investment cash flows, as well as, the expected cash flow values themselves. In this manner, the relative riskiness between different alternatives can be assessed. Analysis techniques also need to be developed that incorporate the principles underlying the CE approach, but avoid its weaknesses in application. One approach would be to rely on simulation and concentrate on estimating the probability distribution for NPV outcomes (where each simulated NPV outcome is evaluated with a risk free rate).\footnote{21} In this manner, the utility function for wealth need only be defined for the current period since now the risk of each investment alternative is defined in present value terms. An expected utility approach could then be used to rank investment alternatives based on the characteristics (e.g., means and variances) of their estimated probability distributions for NPV outcomes.

**CONCLUSION**

This paper began by asking the question; Do risk adjusted discount rates lead to the right investment decisions for landowners? The answer is that a single risk adjusted discount rate, applied to many investment alternatives, will most likely not. However, since the primary purpose of stand level analyses is to provide information regarding the returns to capital allocated to specific sites, the RADR approach may still be useful in providing information for the valuation of risky investments. One simple improvement in the RADR approach would be to have sensitivity analyses around the discount rate separate out the assumptions regarding the risk free and risk premium components. But ultimately, analysis techniques that incorporate risk in investment evaluation without relying on a risk adjustment in the discount rate need to be further developed. In the meantime, it is hoped that recognizing the deficiencies associated with the RADR approach will lead to improved investment guidelines for the landowner.

\footnote{21}{We are indebted to Dr. Kerry Livengood, Department of Forestry, Louisiana Tech, Ruston, Louisiana for bringing this sensitivity analysis format to our attention.}

\footnote{22}{Chambers et al. (1986) and Anderson et al. (1987) provide some examples of this approach.}
LITERATURE CITED


