ECONOMIC MODELS FOR EVALUATING PLANTATION AND UNEVEN-AGED FORESTRY

Robert G. Haight

Abstract.—In the absence of field experience, stand growth models may be used to estimate the yields associated with alternative stand management systems. This paper presents an economic optimization model that may be used in conjunction with diameter-class or single-tree simulators to evaluate stand-level timber management. A discrete-time optimal-control model for determining diameter-class harvest levels and planting intensities over time is formulated without constraints on stand age or size structure. The classical formulations of even-aged and uneven-aged management arc subsets of this general problem. The model is used to address two perennial problems in timber management: (1) estimating the relative economic efficiency of even-aged and uneven-aged management, and (2) estimating the economic costs of steady-state constraints on harvesting uneven-aged stands. In addition, the model is extended to cases where stumpage price is a stochastic process. Adaptive harvesting policies obtained by solving the stochastic formulation provide significantly higher returns than do harvest regimes that do not take period-to-period variation in stumpage price into account.

INTRODUCTION

Forest managers define silvicultural policy for existing and future stands by selecting one of two systems for long-term timber production: even-aged or uneven-aged management. The selection of one of these systems is often based on their relative timber production efficiency, the ability of the systems to meet competing demands for high-quality wildlife habitat and recreational opportunities, and the silvicultural limitations of the tree species involved. In many cases the outputs of alternative systems are estimated by subjective extrapolation of limited field experiments. The difficulties of field experimentation, however, generally preclude an exhaustive examination of a broad range of harvest policies. Thus, the experimental approach alone cannot provide certainty that the best harvest sequence has been tested.

A modeling approach involving a stand growth and yield simulator coupled with a financial optimization algorithm may aid the decision process by providing estimates of optimal harvest regimes for specified management objectives and constraints. Optimal management regimes provide baselines for evaluating regimes based on field experiments. Optimization provides a means to estimate the impacts of changes in uncertain biological and economic parameters on management system efficiency. Finally, the economic returns associated with optimal management regimes may be compared with the returns from alternative uses of timberland and capital to determine the most efficient land use.


2The author is Principal Economist, USDA Forest Service, Southeastern Forest Experiment Station, Box 12254, Research Triangle Park, NC 27709.
Literature in forest management has addressed the problem of evaluating and comparing the efficiency of even-aged and uneven-aged management systems, and comparisons are usually made on the basis of simulated yields from steady-state management regimes (i.e., a repeated sequence of even-aged stands managed with the clearcut system and a repeated sequence of single-tree selection harvests from a sustainable, uneven-aged diameter distribution). For example, using a simulator for Wisconsin northern hardwoods, Hasse and Ek (1981) compared mean annual increments measured in various physical units for even-aged stands with the corresponding yields from steady-state, uneven-aged management regimes. Chang (1981) computed a steady-state, uneven-aged management regime that maximized land expectation value (LEV), and he compared the LEV associated with uneven-aged management to LEVs computed for alternative plantation regimes.

These analyses did not consider the more general problem of managing a given acre of forest land that is currently occupied by an existing stand. For even-aged management this problem may be split into conversion and plantation components: determining the timing and intensity of silvicultural treatments for the current stand and determining the time when the stand is clearcut and replaced with a plantation, and then determining the timing and intensity of silvicultural treatments and clearcut age for the plantation. For uneven-aged management the problem involves determining the sequence of selection harvests that converts the current stand to steady-state, uneven-aged management.

The first section of this paper reviews a general stand investment model that allows the comparison of management regimes that fit these definitions of even-aged and uneven-aged management. The investment model, first described by Haight (1987), is structured to find the sequences of diameter-class harvesting rates and planting intensities that maximize the present value of an existing stand over an infinite time horizon. By placing constraints on planting and harvesting, optimal even-aged and uneven-aged management regimes can be obtained, and measures of economic efficiency can be compared (Sections 2 and 3). Further, the investment model includes the problems of converting an existing stand to plantation management and converting the stand to steady-state uneven-aged management as special cases. As a consequence, the model allows the determination of a management regime that has a present value at least as great as the present values of regimes that fit these constrained definitions of stand management. This "global" optimum can be used to estimate the costs of converting to plantation manage-

ment or converting to steady-state, uneven-aged management.

The fourth section extends the stand investment model to consider cases where stumpage price is a stochastic process. This is an extremely important formulation because in many regions stumpage price varies considerably from year to year. Results from these stochastic models have shown that harvest policies that take price variability into account provide significantly higher returns than do policies that ignore price variance.

INFINITE-TIME-HORIZON HARVESTING

The framework for evaluating even-aged and uneven-aged timber management systems must include a stand growth and yield model that allows harvesting and growth projection of trees in different age and/or size classes. Here I formulate a stage-structured model, which is an extension of age-structured Leslie matrix models where age classes are replaced by growth stages and where transition between stages is incomplete; that is, diagonal elements appear in the growth matrix. Further, the model uses density-dependent, nonlinear functions for predicting regeneration, growth, and survival. A full description of the model is given by Haight and Getz (1987). 3

The descriptive measure of a stand is a frequency distribution of tree diameters. Trees are separated into growth stages represented by 2-inch diameter classes; trees in each growth stage are further described with an average height, cross-sectional crown area, stem volume, and value. For notational purposes, define the n-dimensional vector \( x(t) \) as the stand diameter distribution at the beginning of time period \( t \) before harvesting, and let \( h(t) \) be an \( n \)-dimensional vector representing the numbers of trees harvested from the diameter classes at the beginning of period \( t \). To be feasible, the diameter class harvest levels are constrained

\[
0 \leq h_i(t) \leq x_i(t), \quad i = 1, \ldots, n, \quad t = 0, 1, \ldots
\]

The stage-structured model characterizes stand growth as a transformation of the diameter distribution after harvest \( x(t) - h(t) \) using an \( n \)-dimensional matrix \( G \). Each diagonal element of \( G \) is the product of scalar functions for the proportions of the trees in diameter class \( i \) at time \( t \) that survive the period \((t, t+1]\) and grow into

3Although numerical results described here were obtained using stage-structured models, all of the optimization formulations may be solved using single-tree simulators (see, for example, Roise 1986, Haight and Monsen 1990).
the next diameter class at time $t+1$. Each off-diagonal element represents the proportion of trees that survive and stay in the same diameter class. Tree survival and growth are inversely related to measures of stand density $y(t)$, which are weighted sums of the number of trees in each diameter class after harvesting.

In addition to the growth matrix, there is an input vector $f$ representing the number of trees entering the smallest diameter class through artificial or natural regeneration. Natural regeneration is directly related to the number of seed-bearing trees, and seedling survival is inversely related to stand density. The number of seedlings planted at the beginning of each period is a control variable $s(t)$ with cost $C[s(t)]$.

The diameter distribution of the initial stand $x(0)$ is projected over an infinite horizon with the vector equation,

$$x(t+1) = G[y(t)](x(t) - h(t)) + f[y(t)], \quad t = 0, 1, \ldots$$

(2)

Let the scalar $p(t)$ be the stumpage price per unit volume at the beginning of period $t$, which is independent of tree size and the amount of volume harvested and constant over time:

$$p(t + 1) = p(t), \quad t = 0, 1, \ldots$$

(3)

Letting $v_i$ be the average stem volume in diameter class $i$, the harvest revenue in period $t$ is

$$R[h(t), p(t)] = p(t) \sum_{i=1}^{n} v_i h_i(t).$$

(4)

Assuming that the decision maker's real, risk-free discount rate is $r$ and defining the discount factor as $\delta = 1/(1 + r)$, the infinite-time-horizon problem is to determine harvest and planting levels $h(t)$ and $s(t)$, $t = 0, 1, \ldots$, that maximize the present value $J$ of a stand with initial state $x(0)$,

$$\max_{\{s(t), h(t), t=0,1,\ldots\}} J[x(0)] = \sum_{t=0}^{\infty} \delta^t \{ R[h(t), p(t)] - C[s(t)] \},$$

subject to the feasibility constraints (1), the stand growth equation (2), and the price equation (3).

**UNEVEN-AGED MANAGEMENT WITH STEADY-STATE CONSTRAINTS**

Much of the literature on uneven-aged management has focused on the determination of optimal steady-state management policies. The focus on steady-state management results from the traditional forestry goal of achieving a sustainable yield of timber products. This section examines the role of the steady state in the infinite-time-horizon problem (5), contrasts different methods for imposing steady-state constraints, and deduces the impacts of achieving steady states that are determined with different criteria. In the context of problem (5), the planting controls are set to zero in each time period, and a steady-state constraint is imposed after a transition period with length $T$. Dynamic harvesting problems have been formulated and solved with either equilibrium-endpoint or fixed-endpoint constraints.

The equilibrium-endpoint problem involves the determination of transition and steady-state harvest levels with an equilibrium-endpoint constraint that does not require the achievement of a specific target stand structure:

$$\max_{\{h(t), t=0,\ldots,T\}} J_T[x(0)] = \sum_{t=0}^{T-1} \delta^t R[h(t), p(t)]$$

$$+ \delta^T \frac{1 - \delta}{1 - \delta} R[h(T), p(T)]$$

(6)

subject to the feasibility constraints (1), the stand growth equation (2), and the price equation (3) holding for periods $t = 0, \ldots, T - 1$, and the terminal diameter distribution $x(T)$ satisfying the steady-state constraint

$$x(T) = G[y(T)](x(T) - h(T)) + f[y(T)].$$

(7)

Note that the second term on the right side of problem (6) corresponds to $\sum_{t=T}^{\infty} \delta^t R[h(T), p(T)]$. If $J_T^*[x(0)]$ is the value corresponding to the optimal solution to the $T$-horizon, equilibrium-endpoint problem, then $J_T^*[x(0)]$ approximates $J^*[x(0)]$, the value of the infinite-time-horizon problem and converges to it as $T \to \infty$. The difference between $J_T^*[x(0)]$ and $J^*[x(0)]$ can be regarded as the cost associated with the constraint that the system must be in equilibrium for $t \geq T$. The optimal equilibrium pair $\{x(T), h(T)\}$ depends on $x(0)$, $T$, and $\delta$.

Fixed-endpoint problems involve the determination of a target steady state and a transition regime that reaches the target after a finite transition period. One choice for the target is the extremal steady state (ESS) associated with the infinite-time-horizon problem (5). When stand growth dynamics are nonlinear, optimal harvesting may converge asymptotically to an extremal steady-state pair $\{x_*, h_*\}$ that depends on the discount.

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4Equation (3) could be generalized to include positive or negative price trends, however, this would nullify the formulations and results obtained in the next two sections, which depend on constant prices. This deterministic price equation is included here to contrast with the stochastic price equation presented in Section 4.
factor $\delta$, satisfies the steady-state condition

$$x = G[y](x - h) + f[y],$$

and satisfies a set of necessary conditions derived from Pontryagin's Maximum Principle (see Haight and Getz 1987). If an extremal steady state exists, it can be used in the fixed-endpoint problem as follows:

$$\max_{(h(t), t=0,\ldots,T-1)} I_T[x(0)] = \sum_{t=0}^{T-1} \delta^t R[h(t), p(t)] + \frac{\delta T}{1-\delta} R[h_s, p(T)]$$

subject to the feasibility constraints (1), the stand growth equation (2), and the price equation (3) holding for periods $t = 0, \ldots, T-1$, and the terminal state $x(T) = x_s$. If $I_T[x(0)]$ is the value corresponding to the optimal solution to the $T$-horizon fixed-endpoint problem, then $I_T[x(0)]$ approximates $J^*[x(0)]$, the value of the infinite-time-horizon problem, and approaches it as $T \to \infty$. The additional constraints on $x(T)$ imply that $I_T[x(0)]$ is less than or equal to $J^*[x(0)]$ for any finite $T$. Associated with the fixed-endpoint problem is the question of reachability of the target set (that is, do harvest levels exist that drive the stand to the specified endpoint $x(T)$ during horizon $T$).

A second choice for the fixed endpoint is an investment efficient steady state (IESS), which has been advocated by a generation of forest economists (Duer and Bond 1952; Adams and Ek 1974; Adams 1976; Buongiorno and Michie 1980; Chang 1981; Hall 1983; Bare and Opalach 1987). Investment efficient steady states satisfy an economic stocking criterion that equates the marginal value growth percent of the stand to the discount rate. Let the pair $(x_h, h)$ be a steady-state solution satisfying condition (8) for some choice of $h$. Investment efficient steady states are determined independently of the transition regime by solving a maximization problem involving the present value of the steady-state pair (that is, the term $\frac{\delta}{1-\delta} R[h, p]$) and a term $R[x_h - h, p]$ representing the opportunity cost of the residual growing stock. The opportunity cost is the revenue that could be obtained by clearcutting the residual steady-state growing stock $x_h - h$. The maximization problem is

$$\max_h \left[ \frac{\delta}{1-\delta} R[h, p] - R[x_h - h, p] \right]$$

subject to steady-state constraint (8). This formulation has appealed to forest economists because if the growing stock $x_h - h$ is viewed as a capital investment, then (10) is equivalent to maximizing land expectation value (LEV), as defined by the Faustmann formula (Chang 1981). The optimal solution value for problem (10) is then used for comparison with the LEV associated with even-aged management (Chang 1981). It turns out that an investment-efficient steady state $(\bar{x}, \bar{h})$ that solves problem (10) satisfies conditions for the extremal steady state only in the special case where the revenue function $R$ is a linear combination of the harvest level and $\bar{h} > 0$ (see Haight 1985 for details). Thus, a solution to the fixed-endpoint problem (9) that is constrained to achieve an investment efficient steady state has, in most cases, a lower present value relative to the present value of the solution to the fixed-endpoint problem that is constrained to achieve the extremal steady state.

A third choice for the fixed endpoint is the maximum-sustainable-rent (MSR) steady state that solves the problem

$$\max_h R[h, p]$$

subject to steady-state constraint (8). Note that when the revenue $R$ is defined as harvest yield, the solution to problem (11) is a maximum-sustainable-yield steady state. If $(\bar{x}, \bar{h})$ is a maximum-sustainable-rent solution to problem (11), then $(\bar{x}, \bar{h})$ approaches the extremal steady-state solution to the infinite-time-horizon problem (5) as $\delta \to 1$ (see Haight and Getz 1987). Thus, for $\delta < 1$, a solution to the fixed-endpoint problem (9) that is constrained to achieve a maximum-sustainable-rent steady state has a lower present value relative to the present value of the solution to the fixed-endpoint problem that is constrained to reach the extremal steady state.

Numerical optimization studies have been performed to compare the results from fixed and equilibrium endpoint problems under various initial stand conditions, lengths of planning horizons and discount rates using a stage-structured model for California white fir stands (Haight and Getz 1987). As expected, for a given transition period, the solution to the equilibrium-endpoint problem has a higher present value than the solution to any fixed-endpoint problem, since the equilibrium-endpoint formulation places fewer constraints on the terminal steady state. The equilibrium-endpoint policy depends on the initial stand structure and transition period length, and it may differ in terms of species composition and sustainable harvest value from ESS, IESS and MSR structures. As the transition period lengthens, the equilibrium-endpoint policy approaches the ESS, and the cost of the terminal steady-state constraint approaches zero. Numerical solutions showed that the cost of the terminal steady-state constraint can be large (greater than 10% of the present value of
The cost of a fixed steady-state constraint depends on the criterion used to determine the target steady state and the transition period length. With an ESS target, the solution to the fixed-endpoint problem approaches the solution to the equilibrium-endpoint problem as the transition period lengthens, and the cost of steady-state constraint approaches zero. Numerical results showed that the cost of the ESS target in short transition periods can be large. The costs of converting to steady states in short time periods was higher with low discount rates, and in initially overexploited stands, they amounted to 60% or more of the present value of unconstrained management. The costs of achieving the IESS or the MSR policies can be severe (greater than 60% of the present value of the unconstrained solution) regardless of the transition period length. As the discount rate approaches zero, the ESS policy approaches the MSR policy, and the cost of achieving the MSR policy approaches zero.

The value of the steady-state yield depends on the criterion used to determine the steady-state target. The MSR policy always provides the highest value yield, and those interested in attaining the highest value yield would want to convert to the MSR policy. In previous studies, the objective associated with the achievement of an IESS structure is the maximization of the present value of harvested yields during the transition to a steady state. It turns out that the IESS structure has the highest investment value (i.e., present-value of steady-state yields net investment cost), but this criterion is not consistent with the objective of maximizing present value. In fact, this objective can be achieved more efficiently by solving the equilibrium-endpoint problem. The ESS policy may produce a relatively low sustainable yield, but this is offset by the low cost associated with achieving the ESS. The ESS has the property that, once achieved, there exists no transition policy away from the ESS that improves the present-value of harvesting.

**EVEN-AGED MANAGEMENT**

Numerical studies of even-aged stand management have either solved for the optimal thinning regime for an existing stand that terminates with clearcut and initiation of plantation management (e.g., Roise 1986), or solved for the optimal infinite-series plantation regime (e.g., Haigh et al. 1985). These problems can be solved simultaneously by constraining the harvest and planting controls so that the infinite-time-horizon problem (5) is converted into finite-time-horizon conversion and plantation problems.

By definition, a plantation is established after harvest at the beginning of period $K$ and is managed with a thinning regime during an $L$-period rotation. For planting density $s(K)$ and harvest levels $h(t)$, $t = K + 1, \ldots, K + L$, define $Q_{K,L}$ as the present value of a single-rotation plantation regime

$$Q_{K,L} = -\delta^K C [s(K)] + \sum_{t = K+1}^{K+L} \delta^t R [h(t), p(t)], \quad (12)$$

subject to the growth equation (2) defined over $t = K, \ldots, K + L$ and the clearcut constraint at rotation age $h(K + L) = x(K + L)$. Note that the planting control is zero except for the beginning of period $K$ when the plantation is established. If we assume that an $L$-period plantation regime is applied repeatedly in perpetuity, the present value of the infinite series of plantations established at the beginning of period $K$ is $Q_{K,L}/(1 - \delta^L)$, which is consistent with the assumption that prices, costs and interest rates are constant over an infinite planning horizon.

With this definition of plantation management, the problem of converting an existing stand in $K$ periods to an infinite series of $L$-period plantations is the $K + L$ period problem,

$$\max_{\{h(t), t = 0, \ldots, K + L, s(K)\}} J_{K,L}[x(0)] = \sum_{t = 0}^{K} \delta^t R [h(t), p(t)]$$

$$+ \frac{Q_{K,L}}{1 - \delta^L}, \quad (13)$$

subject to the growth equation (2) defined over $t = 0, \ldots, K + L$ and clearcut constraints at the beginning of periods $K$ and $K + L$. The planting control is zero during the conversion regime, and it is positive at the beginning of period $K$ as defined in the plantation regime (equation 12). Also note that, when $K = 0$, $h(0) = x(0)$, and problem (13) reduces to the plantation management problem.

Denote $J_{K,L}$ as the optimal solution to the conversion and plantation problem (13) for a given conversion period $K$ and rotation age $L$. The best conversion period length and rotation age is found by maximizing $J_{K,L}$ over all non-negative $K$ and $L$. This problem is simplified by noting that, by the principle of optimality, the plantation management problem can be solved separately from the conversion problem. Because problem (13) states that the stand is clearcut at the beginning of period $K$, the stand structures $x(t)$, $t = K + 1, \ldots, K + L$, depend on the planting density $s(K)$ and are independent of the conversion regime prior to
period $K$. Thus, $J_{K,L}$ is the sum of two maximization problems:

$$
\max_{\{h(t), t=0, ..., K\}} \sum_{t=0}^{K} \delta^t R[h(t), p(t)]
$$

subject to the growth equation (2) defined over $t = 0, \ldots, K$ and the clearcut constraint at the beginning of period $K$; and

$$
\max_{\{a(K); h(t), t=K+1, \ldots, K+L\}} \frac{Q_{K,L}}{1 - \delta^L}
$$

subject to the growth equation (2) defined over $t = K, \ldots, K + L$ and the clearcut constraint at the beginning of period $K + L$. Further, the plantation management problem (15) can be solved for any rotation age $L$ independently of the $K$-period conversion problem (14). This result holds for any conversion length $K$.

Define $Q^*_L$ as the present value of the optimal infinite-series plantation regime that is obtained by solving problem (16) for rotation length $L$. Define $Q^*$ as the maximum of $Q^*_L$, $L = 1, 2, \ldots$. The $K$-period conversion problem depends on $Q^*$ as follows:

$$
J_K^* = \max_{\{h(t), t=0, ..., K\}} \sum_{t=0}^{K} \delta^t R[h(t), p(t)] + \frac{Q^*}{\delta^K}.
$$

Since $Q^*/\delta^K$ is constant for a given $K$, the optimal solution value $J_K^*$ to problem (16) is independent of $Q^*$. If $J^*$ is the value of the optimal conversion regime over all $K$ (that is, $J^*$ is the maximum of $J_K^*$, $K = 0, \ldots, L$), $J_K^*$ includes the term $Q^*/\delta^K$, and the value $J^*$ depends on $Q^*/\delta^K$.

In summary, the problem of determining the optimal conversion and plantation management regime is a special case of the infinite-time-horizon problem (5); where, in this special case, constraints are imposed to require a clearcut and switch to plantation management. Further, the plantation problem is independent of the conversion problem and can be solved separately. The optimal conversion period length and conversion harvest regime depend on the value of the optimal plantation regime.

The problem of valuing land and existing trees dates back to Faustmann’s (1849) work on forest land valuation. Assuming that even-aged management would be practiced indefinitely, Faustmann defined forest value as the sum of the stand value, which is the present value of harvests taken during the conversion period, and land expectation value (LEV), which is the present value of an infinite-series of plantations. Thus $J_{K,L}$, the present value of conversion and plantation harvesting in expression (13), is equivalent to Faustmann’s definition of forest value, and $Q^*_L$, the present value of the optimal plantation regime that solves problem (15), is equivalent in definition to the maximum LEV for a repeated sequence of plantations with rotation $L$. Faustmann noted that, since plantation management starting with bare land would eventually be practiced in perpetuity, LEV is independent and separable from the present value of the conversion regime. Thus while this result is discussed above in a more rigorous manner, it was intuitively obvious to Faustmann over 100 years ago.

It should be emphasized that the separability of the plantation problem depends on the key assumptions that prices, costs, and interest rates are constant over an infinite time horizon. If the assumptions of infinite time horizon or constant economic parameters are relaxed, optimal plantation rotations will vary over time (see Hardie et al. 1984), and will depend on the period in which the conversion to plantation management takes place.

Numerical solutions to even-aged and uneven-aged management problems have been computed for ponderosa pine stands in Arizona (Haight 1987), white fir in California (Getz and Haight 1989), and mixed-conifer stands in Idaho (Haight and Monsrud 1990). In all of these cases models predict abundant natural regeneration, and the relative efficiencies of the two management systems depends to a large degree on the initial stand structure. When the stand is young and vigorous selection harvesting and uneven-aged management is optimal. On the other hand, converting mature stands to vigorous even-aged stands via natural or artificial regeneration is optimal.

**UNEVEN-AGED MANAGEMENT WITH STOCHASTIC PRICES**

Traditional formulations of timber harvesting problems, including those discussed above, assume that forecasts of stumpage price and stand growth are known with certainty over long time horizons. A clear violation of the certainty assumption is the unpredictable variation in stumpage price that occurs from time to time. Results from these deterministic optimization problems are called open-loop harvest strategies because they ignore unexpected changes in the economic or biological systems, and they are applied as though the original forecasts are correct. Open-loop policies are not realistic in situations where there are opportunities for monitoring the stand and market state as management proceeds.

Recognizing that uncertainty exists in economic forecasts, recent studies have employed stochastic optimization to design adaptive timber management strategies that cope with unpredictable stumpage price variation. An adaptive decision strategy is a conditional program
of actions where management activity at each decision point is a function of the stand and market state at that point. Adaptive decisions are sensible when future stumpage price cannot be predicted with certainty, but at any decision point, the current stand and market price can be observed and used to make a harvest decision.

Adaptive management strategies have been developed for clearcutting even-aged stands (Norström 1975, Lohmander 1987, Brazee and Mendelsohn 1988) and for thinning uneven-aged stands (Kaya and Buongiorno 1997, 1989, Haight 1990a). For both types of management, timber harvesting is timed to market needs using the simple rule of “don’t cut when prices are low.” For even-aged management, the stand is clearcut whenever the observed stumpage price is greater than an optimal reservation price. For uneven-aged management, harvest intensity is directly related to the product of the observed stumpage price and the stand stocking level. Such policies provide significantly higher returns than do cutting practices that are based on average prices alone.

Stochastic management studies have in common the assumption that timber management is the preferred land use and that either clearcutting and plantation management or selection harvesting and uneven-aged management is the preferred system in perpetuity. Clearly this assumption does not hold in all cases. Some landowners may want the option to switch from timber management to another land use (e.g., agriculture) at any point in time. Other landowners may want the option to switch from selection harvesting to plantation management.

This section formulates an adaptive management function that prescribes harvest intensities and shifts between land uses and management systems under the assumption of stochastic stumpage prices. I assume that the size of the forest investment represents only a small fraction of the total wealth of the investor, and forestry returns only marginally influence the investor’s overall utility. Thus, any risk-aversion is ignored, and the management objective is to determine the adaptive harvest strategy that maximizes the expected present value of the existing stand.

Building on the feedback thinning function for uneven-aged management developed by Haight (1990a), I develop a three-parameter adaptive management function for thinning and clearcutting an existing stand. The function maps the relationship between harvest intensity (trees/acre) and observed stand value ($/acre). The first two parameters define the slope and location of a thinning function. The slope parameter is positive so that thinning intensity increases with stand value. The location parameter is the stand reservation value, which is the minimum stand value required for thinning to take place. The third parameter, called maximum stand value, is the stand value at which clearcutting is prescribed.

This adaptive management function may be used to solve two management problems. The first problem involves determining when to switch from forestry to another land use. The objective is to maximize the expected present value of revenue obtained from thinning and clearcutting an existing stand and from switching land use. The revenue obtained when the stand is clearcut is the sum of the value of the trees and the value of the bare land. Land values are constant and known with certainty, and they represent the present value in perpetuity of the alternative land use. The second problem is to determine when to switch from selection harvesting to plantation forestry. The problem is a special case of the first problem where the value of bare land is the expected present value of a perpetual sequence of plantation regimes.

Stumpage price is modeled as a stationary random process. The expected price $\bar{p}$ represents the dollars per thousand board feet of harvested timber ($/Mbf$) and is constant over time. The observed price, $p(t)$, $t = 0, 1, \ldots$, is the sum of the expected price and a random error term and is the stochastic equivalent of $p(t)$ in equation (3). The stochastic price model is

$$p(t) = \bar{p} + \epsilon(t), \quad t = 0, 1, \ldots$$

where $\epsilon(t)$, $t = 0, \ldots, T$, are independent and identically distributed $N(0, \sigma^2)$. 5

The stochastic price equation is used to compute harvest revenue. The revenue in period $t$ is now a random variable defined by

$$R[h(t), p(t)] = p(t) \sum_{i=1}^{n} v_i h_i(t).$$

In period $T$ the stand is clearcut. Let $L$ be the bare land value, which may represent either the value of the land in a non-forestry use or the value of an infinite series of plantations if the land is used for even-aged forestry. The terminal value function is

$$S[x(T), p(T)] = p(T) \sum_{i=1}^{n} v_i x_i(T) + L.$$  

5This representation ignores any autocorrelation or trends in the stumpage price forecast. Nevertheless, the form of the model is not unrealistic because it has been chosen as the best model for stumpage price behavior in other optimization studies of timber harvesting (Kaya and Buongiorno 1987, Brazee and Mendelsohn 1988).
The adaptive harvest policy specifies three kinds of management (no action, thinning, or clearcutting) depending on the observed stumpage price and stand stock. This is achieved with a three-parameter feedback function that maps the relationship between the total number of trees harvested \( H(t) \) and the observed stand value \( R[x(t), p(t)] \), which is computed by substituting \( x(t) \) for \( h(t) \) in equation (18). The adaptive management function is,

\[
H(t) = \begin{cases} 
0 & \text{if } R[x(t), p(t)] \leq b_2 \\
 b_1 \{ R[x(t), p(t)] - b_2 \} & \text{if } b_2 < R[x(t), p(t)] < b_3 \\
 \sum_{i=1}^{n} x_i(t) & \text{if } R[x(t), p(t)] \geq b_3
\end{cases}
\]

(20)

where \( b_1, b_3 > 0 \), and \( b_2 \) is unrestricted.

The parameter \( b_2 \) is the stand reservation value; no harvest is taken when the observed stand value is less than \( b_2 \). Stand reservation value is analogous to “reservation price,” which is used to determine the rotation age for even-aged stands with stochastic prices (see Brazee and Mendelsohn 1988). The important difference in equation (20) is that when the stand reservation value is exceeded, harvest intensity is an increasing function of stand value rather than a “pulse” harvest as in the even-aged management case.

When the stand reservation value is exceeded, the harvest level is a linear function of the observed stand value with positive slope \( b_1 \). This is consistent with Lohmander’s (1987) general result that harvest intensity increases with stand value. In this range of stand values, a harvest takes a portion of the trees from the stand and represents thinning. Because the harvest level is defined as the total number of trees cut, it must be transformed into diameter class harvest levels \( h(t) \) according to some rule. In this formulation, thinnings take the largest trees first. This rule is consistent with the optimal diameter-class thinning behavior found by Kaya and Buongiorno (1987) with a Markov decision model of uneven-aged management with stochastic stumpage prices.

When the observed stand value is greater than the maximum stand value parameter \( b_3 \), the stand is clearcut and management ends. Thus, the horizon \( T \) is a random variable that depends on the stochastic stumpage prices and the maximum stand value parameter.

The optimization problem is to determine the parameters \( b_1, b_2, \) and \( b_3 \) of the feedback harvest equation (20) that maximize the expected present value \( \mathcal{J} \) from

\[
\max_{\{b_1, b_2, b_3\}} \mathcal{J} = \mathcal{E} \left\{ \sum_{t=0}^{T-1} \delta^t [R(h(t), p(t)) + \delta^T S[x(T), p(T)] \right\},
\]

(21)

subject to the feasibility constraints (1), the stand growth equation (2), and the price equation (17). Note that the horizon \( T \) represents the period in which the stand is clearcut: \( T = \{ t | R(x(t), p(t)) \geq b_3 \} \) (see equation 20). When the stand is clearcut, the terminal value function \( S[x(T), p(T)] \) is the sum of the clearcut value of the stand and the bare land value (see equation 19).

Application of this adaptive management model to test cases for California white fir (Haight 1990b) show that, for any land value, the expected present value of optimal management is directly related to the level of price variation (i.e., \( \sigma^2 \) associated with \( \epsilon \) in the price equation 17). Furthermore, the cost of ignoring price variation may be substantial. Thus, landowners who are willing to employ adaptive management strategies and to accept an uneven flow of revenue may significantly improve their net present value in the presence of price variation. These results are consistent with stochastic stand management studies in which either even-aged or uneven-aged management is applied in perpetuity (Norström 1975, Lohmander 1987, Brazee and Mendelsohn 1988, Kaya and Buongiorno 1989, Haight 1990a).

When the question involves when to switch from forestry to an alternative land use, the longevity of timber management depends on bare land value. For any level of price variation, the expected time of clearcut is inversely related to land value. The effect of price variation on optimal time of clearcut is not independent of land value. A surprising result is that, for relatively low land values, increasing price variation may decrease the expected time of clearcut. Without price variation, perpetual selection harvesting is optimal. As price variation increases, it is better to take advantage of an unusually high stumpage price by clearcutting and accepting the low land value. For relatively high land values, the effect of price variation is reversed. Without price variation, the land value is greater than returns from timber management and thus the stand is immediately clearcut. With price variation it is better to postpone clearcutting until a high stumpage price is offered.

When the question involves the kind of management system to employ, the size distribution of the initial stand rather than price variation plays the critical role in determining the relative efficiency of plantation versus uneven-aged forestry. If the existing stand has a large number of young, fast-growing trees, selection harvesting is optimal with and without price variation. On the other hand, it is better to clearcut mature stands...
and establish plantations, regardless of the level of price variation (although the exact time of clearcut depends on the observed stumpage price).

It would be fruitful to extend this formulation to cases where the management objective includes risk aversion. A complete analysis must determine optimal harvest levels simultaneously with consumption levels given specific assumptions about capital market function. It would also be interesting to determine the relative effects of biological and economic risk and thus establish which system should be monitored more closely. The methods presented here are a good starting point for developing adaptive management formulations for both these problems.

CONCLUSIONS

The above analysis has shown that the infinite-time-horizon harvesting model (equation 5) includes even-aged and uneven-aged harvesting problems as special cases. These special cases arise when constraints are imposed to require a clearcut and switch to plantation management after a specified time period or to achieve a steady-state, uneven-aged management regime after one or more harvests. The present values of these even-aged and uneven-aged management regimes are called forest values, after the definition that Faustmann originally assigned to the present value of converting a stand to plantation management. Forest values for different management regimes can be compared to determine the most efficient timber management system.

Land expectation value is the present value of an infinite series of plantations, starting with bare ground. Plantation regimes that maximize LEV are independent of the existing stand and the harvesting regime that is used before the stand is clearcut. Land expectation value has no equivalent measure in uneven-aged management. To compare the economic efficiencies of the even-aged and uneven-aged management systems applied to a given stand, the present values of conversion and plantation regimes for even-aged management should be compared with the present values of transition and steady-state regimes for uneven-aged management.

The present values of solutions to the infinite-time-horizon problem (5) have the highest present values. Solutions that maximize the present value of converting to plantation management or converting to steady-state, uneven-aged management are constrained and therefore have present values that are less than or equal to the present values of solutions to the general management problem. The resemblance of management regimes obtained by solving problem (5) to either even-aged or uneven-aged management depends on the initial stand structure, in addition to the biological and economic models and parameters used in the analysis. Where maximum efficiency is the guiding objective, the evaluation of timber management systems should include a solution to the general stand management problem for each kind of stand that exists on the forest.

Recent optimization studies have begun to analyze the effects of stochastic stumpage prices on harvesting system efficiency. The most surprising result is that increasing price variability may increase the expected profits from timber management provided the appropriate adaptive management strategy is adopted. Policies that do not use price variation in the determination of harvest intensity provide significantly smaller returns. This result shows the importance of optimization formulations that include the stochastic components of the management system.

LITERATURE CITED


