A DYNAMIC PROGRAMMING OPTIMIZATION MODEL
FOR UNEVEN-AGED LOBLOLLY-SHORLEAF
PINE STANDS IN THE MID-SOUTH 1/

James E. Horvedt and Keith B. Ward 2/

Abstract.--A dynamic programming model was developed for uneven-aged loblolly-shortleaf pine stands typical of the
Mid-South. The model was developed to address two different
long-term management goals: to optimize the conversion of an
initial uneven-aged stand structure to a different, more
desirable stand structure and to determine the optimal,
infinite-horizon, equilibrium steady-state stand structure
for uneven-aged loblolly-shortleaf pine stands given some
initial uneven-aged stand structure.

INTRODUCTION

The uneven-aged silvicultural system will
become more significant as environmental
pressures, the protection of endangered species,
and other amenity value interests become more
important and influence the way that forest lands
are managed. The advantages of the system over
more intensive even-aged management systems are
broad in scope. Uneven-aged management can
provide an improvement in stocking and growth of
desirable tree species, an attractive forest
environment for recreation, and a suitable
habitat for diversified wildlife populations
(Baker and Harrington 1980). Further, it can
provide an alternative to landowners who possess
few acres, who can afford little to no capital
investment, who want to avoid costly site
preparation, and who desire a flow of periodic
income without interruption of stand regeneration
(Williston 1978). Baker (1985) reports that the
selection system is successfully being used to
manage approximately a million acres of
industrial lands and over a million acres of
nonindustrial private lands in the South today.
Wildlife and other amenity value and
environmental issues will more than likely cause
a shift toward uneven-aged management on a large
area of public forest lands in the South as well.

Managing uneven-aged stands will probably
require a greater amount of forestry expertise
and incite than has been required in managing
even-aged stands. Although not wanting to imply
that even-aged management is without its
challenges, management decisionmaking associated
with harvesting even-aged stands is generally
relatively simple: at the end of a rotation, a
stand is clearcut and is either naturally or
artificially regenerated. In contrast, when an
uneven-aged stand is cut at the end of a cutting
cycle, decisions must be made concerning the
structure of the residual stand. Furthermore,
timber markers must take care to mark the stand
so that the desired structure is ensured,
measures must be taken to ensure adequate
regeneration, and harvesting operations must be
designed to minimize damage to the residual
stand.

In a related problem, to convert to a desired
even-aged structure, an existing stand is
clearcut at some point in time. The stand is
then regenerated at a density which is expected
to yield the appropriate structure at the desired
rotation age. When converting a stand to an
even-aged management system, the major decisions
to consider are the timing of the harvest of the
existing stand and regeneration considerations.
For an uneven-aged stand, the conversion of an
existing stand to the desired structure may not
be feasible with a single harvest. Indeed, an
extended conversion period may be necessary to
gradually achieve the desired stand structure. Consequently, the conversion to a desired uneven-aged stand structure involves not only decisions concerning the timing and intensity of harvesting and management practices on the existing stand but also decisions on the timing and intensities of all subsequent cuttings and management practices until the desired structure is attained.

The most common regulation method proposed for uneven-aged pine stands is the "basal area-maximum DBH-Q" (BDQ) technique. Under the BDQ method, these decisions are based on desired residual stand structures after a harvest. Under the BDQ technique, residual growing stock, diameter distribution, maximum-sized tree, and cutting cycle goals must be chosen (fig. 1). The residual growing stock goal is the growing stock left immediately after a harvest cut and is often expressed in terms of basal area per acre. The diameter distribution of a managed uneven-aged stand has an inverse J-shape which is often ideally characterized by Q, the ratio of the number of trees in a diameter class to the number of trees in the next higher class. Q is a constant attribute of a negative exponential function that is often used to describe the diameter distribution of a balanced uneven-aged stand structure. The maximum-sized tree is the tree(s) with the largest DBH left after a harvest. All trees larger than the maximum-sized tree(s) are harvested.

Sawtimber and pulpwood product yields employed in the dynamic programming model were calculated from models for all-aged loblolly-shortleaf pine stands with site indices of 85-95 feet (base age 50 years) (Murphy and Farrar 1982, 1983). Because of the growth and yield model employed in this study, residual merchantable basal area (MB), residual sawtimber basal area (SB) derived from a sawtimber to merchantable basal area ratio (SRAT), and elapsed time (t) were specified as decision variables in the dynamic programming model, rather than the traditional BDQ variables. Consequently, a particular management regime was identified by its unique combination of these three variables.

Although employment of the traditional BDQ decision variables provide more information on how to structure a stand than do the use of MB and SB alone, a given SRAT (the ratio SB/MB), is associated with a number of different D-Q combinations. For example, in figure 2, four different D-Q combinations that correspond to an SRAT of .60 are presented. All are biologically feasible. As a result, the BDQ method can still be utilized indirectly when using the Murphy and Farrar growth and yield models and, consequently, when using the dynamic programming model presented in this paper.

Previous research focusing on the determination of optimal uneven-aged management regimes has concentrated primarily on determining an optimal equilibrium stand structure and cutting cycle (e.g., Chang 1981, Hall 1983, and Hotvedt et al. 1989). Static models have usually been employed in these studies. One of the problems associated with static approaches to determining optimal management regimes for uneven-aged forest stands is that the structures of existing stands do not usually correspond to the optimal structures derived from these models. Consequently, the existing structures must be converted to the optimal structures over a period of time. The static approaches do not generally provide for an optimal conversion strategy, however. In fact, because the conversion period may be long, determining and following an optimal conversion strategy may be more important to a forest manager than achieving the optimal ending equilibrium stand structure and cutting cycle. NIPF owners who view the management of their lands on a short-term basis, in contrast to industry and public agencies who manage for more long-term, corporate goals or amenity goals, may also be more interested in shorter-term conversion strategies than in determining the longer-term equilibrium steady-state stand structure.
Because of the major shortcomings of static approaches, a dynamic programming model was developed for determining optimal management strategies for uneven-aged loblolly-shortleaf pine stands in northern Louisiana and southern Arkansas. The model was designed to determine the optimal strategy for converting an initial uneven-aged stand structure to a different, more desirable target stand structure within a given time horizon. Thus, the optimal management strategy includes the cutting cycle length and conversion strategy. Furthermore, if the planning period is sufficiently long, the model can also determine the optimum equilibrium steady-state stand structure. In a related study, Haight et al. (1985) presented a gradient-based method, another class of dynamic optimization techniques, to determine the optimal sequence of diameter distributions and selection harvests for a given initial stand structure.

**THE DYNAMIC PROGRAMMING MODEL**

Dynamic programming is a mathematical optimization approach used for problems requiring a series of decisions which are sequentially related. In the case of managing uneven-aged stands, stand management decisions made in a prior time period affect decisions that can or might be made in future time periods. A dynamic programming model helps determine the optimal sequence of such decisions.

Regardless of the nature or complexity of the problem, all problems solvable by means of dynamic programming have many common characteristics (Hillier and Lieberman 1980). First, the problem and objective function are capable of being separated into a series of smaller problems or sequential steps. These divisions are called **stages** (see Stage 1 in fig. 3). A decision is required at each stage which **transforms a state** (see S1j in fig. 3) of the system into a state of the succeeding stage. A **state descriptor** is used to identify a state of the system; a complete set of descriptors is a state and there may be many states. A **node** is a state/stage combination; therefore, each node is unique. An **arc** is the path from one node to another. A **transformation function** is required to link consecutive stages together. A particular state of the system is dependent only, on the latest state and decision of the system; a decision at any give stage is independent of the decisions of previous stages. Finally, a **recursive solution procedure** must be possible to allow the optimal decision sequence to be identified. In summary, the objective of a dynamic programming model is to determine the optimal path (series of decisions) from the initial state to an ending state. If the planning horizon is sufficiently long, a dynamic programming model might also identify an optimal equilibrium steady state, such as that illustrated in figure 4.

**Figure 3.** Possible routes from an initial state to an ending state

Two major phases are involved in dynamic programming algorithms. In the first phase, optimal decisions are made at each stage in which states are attained from their respective previous stages. The second phase involves tracing back through the system, using a recursion procedure, to identify the overall optimal sequence of decisions.

In terms of the dynamic programming model in this paper, a stage corresponds to a movement from one stand structure to another stand structure over a period of t years. The state variables are total residual merchantable basal area (MB), sawtimber basal area (SB) derived from the sawtimber basal area/total merchantable basal area ratio (SRAT), and the elapsed time (t) associated with the transition from one state to another. An action or decision is whether or not to harvest, and if so, how much to harvest of sawtimber and pulwood basal area. A return is something which a system generates over a stage of a process, that is, something generated in the transition from one state to another. In the dynamic programming model presented in this paper, a number of return variables can be specified: volume harvested, sawtimber volume harvested, nondiscounted net return, and discounted net return. The value of a particular

**Figure 4.** Possible routes from an initial state to an optimal steady state
Table 1: The general dynamic programming mathematical model for uneven-aged loblolly-shortleaf pine stands

**Recurrence Relation**

\[
\begin{align*}
 f(n,i) &= \max - (r(n,i,k) + f(n-1,j)) \\
 &\quad (MB_n, SB_n, t) \\
 r(n,i,k) &= R_n/(1 + p)^{nt} \\
 \text{where } R_n &= P_1v_1(SB_n, MB_{n-1}) + P_2v_2(SB_n, MB_{n-1}) - C_n
\end{align*}
\]

**Transition Relation**

\[
\begin{align*}
 j &= t(n,i,k), \text{ or} \\
 MB_n &= g_1(MB_{n-1}, t) - HMB_n \\
 SB_n &= g_2(SB_{n-1}, MB_{n-1}, t) - HSB_n
\end{align*}
\]

**Constraints**

\[
\begin{align*}
 l_1 &\leq MB_n \leq l_u & \text{Residual stand constraints} \\
 k_1 &\leq SB_n/MB_n \leq k_u & \\
 MB_n &\leq m_u & \text{Regeneration constraint} \\
 0 &\leq h_1(MB_n, MB_{n-1}) \leq g_1(MB_{n-1}, t) & \text{(ba)} \\
 0 &\leq h_2(SB_n, SB_{n-1}, MB_n, MB_{n-1}) \leq g_2(SB_{n-1}, MB_{n-1}, t) & \\
 v_1(SB_n, MB_{n-1}, MB_n, MB_{n-1}) &\geq v_1 & \text{Harvest constraint} \\
 t_1 &\leq t_n \leq t_u & \text{Elapsed time (cutting cycle) constraint}
\end{align*}
\]

The model was set up to optimize management decisions over a particular planning horizon based on one of the four return functions or objectives chosen by the user. For simplicity and because it is the primary criterion of interest, the present net value criterion is used in describing the construction of the optimization model.

The problem formulated in a dynamic programming framework is presented in Table 1; variable definitions are presented in Table 2. In dynamic programming, a recurrence relation is used to calculate the optimal values of each
Table 1. The general dynamic programming mathematical model for uneven-aged loblolly-shortleaf pine stands

Recurrence Relation

\[ f(n,i) = \max \{ r(n,i,k) + f(n-1,j) \} \]
\[ (MB_n, SB_n, t) \]

\[ r(n,i,k) = R_n / (1 + p)^{nt} \]

where \( R_n = P_1V_1(SB_n, SB_{n-1}, MB_n, MB_{n-1}) + P_2V_2(SB_n, SB_{n-1}, MB_n, MB_{n-1}) \) - \( C_n \)

Transition Relation

\[ j = t(n,i,k), \text{ or} \]
\[ MB_n = g_1(MB_{n-1}, t) - HMB_n \]
\[ SB_n = g_2(SB_{n-1}, MB_{n-1}, t) - HSB_n \]

Constraints

\[ 1 \leq MB_n \leq l_u \]
\[ k_1 \leq SB_n/MB_n \leq k_u \]
\[ MB_n \leq m_u \]

\[ 0 \leq h_1(MB_n, MB_{n-1}) \leq g_1(MB_{n-1}, t) \]
\[ 0 \leq h_2(SB_n, SB_{n-1}, MB_n, MB_{n-1}) \leq g_2(SB_{n-1}, MB_{n-1}, t) \]

\[ v_1(SB_n, SB_{n-1}, MB_n, MB_{n-1}) \geq v_1 \]
\[ t_1 \leq t_n \leq t_u \]

state is a function (sum) of the returns generated when the system starts in that state and a particular plan or set of actions is followed. In this problem, the value of a state is the sum of the stage returns over some finite planning horizon. A number of return functions can be specified to determine the value of a state: total volume harvested, total sawtimber volume harvested, total nondiscounted net returns generated, and total discounted net returns generated. Inequalities specified in the model above determine the range of the stages, the range of the state variables, and the range of the action variables.

The model was set up to optimize management decisions over a particular planning horizon based on one of the four return functions or objectives chosen by the user. For simplicity and because it is the primary criterion of interest, the present net value criterion is used in describing the construction of the optimization model.

The problem formulated in a dynamic programming framework is presented in table 1; variable definitions are presented in table 2. In dynamic programming, a recurrence relation is used to calculate the optimal values of each
A transition equation, \( t(n,1,k) \), relates a predecessor state in stage \( n-1 \) to the current stage, state, and action variables. Or, it relates the predecessor state \((n-1,j)\) to the current state \((n,1)\) under action \(k\). The transition equation is used to work backwards from the ending state, through the optimal path, to the beginning state to track the optimal solution path or set of decisions and actions. The various parts of the growth and yield model are used in this process.

Four general types of constraints are specified in the dynamic programming model: (1) residual stand constraints which restrict total merchantable basal area and the sawtimber basal area ratio to fall within user-specified upper and lower limits, (2) a regeneration constraint which restricts the total merchantable basal area from increasing beyond a user-specified upper limit, (3) harvest constraints which restrict total merchantable and sawtimber basal area harvested to be less than initial basal area plus growth, (4) harvest constraints which require sawtimber or pulpwood volume harvested to exceed a user-specified minimum level, and (5) elapsed time constraints which require successive harvests to fall within user-specified upper and lower limits.

**EXAMPLES OF USES OF THE DYNAMIC PROGRAMMING MODEL**

**Beginning State Same as Static Optimal**

The first example used for illustrating the use of the dynamic programming model is a case in which the beginning state (stand structure) of an uneven-aged loblolly-shortleaf pine stand is the same as the optimum state derived from a static model (see Hotvedt et al. 1989). In the static model, the optimum combination of MB-SRAT-CC is 55-.55-5. Therefore, the beginning and ending states in the dynamic programming model are specified to have an MB of 55 square feet and an SB of 30 square feet. The diameter distributions of three combinations of D and Q corresponding to an SRAT of .55 are illustrated in figure 5. All combinations are biologically feasible.

As figure 6 illustrates, the optimal combination of MB-SB-CC does not change over time. It is constant at 55-30-5. Over the 5-year cutting cycle, the stand grows to 68 square feet of merchantable basal area and 45 square feet of sawtimber basal area and is cut back to a residual of 55 square feet of merchantable basal area and 30 square feet of sawtimber basal area.

It is readily apparent that if a stand is already being optimally managed according to the recommendations of the static model, then the optimal equilibrium steady-state regime determined by the dynamic programming model will be consistent with the optimal regime determined by the static model. The results of the two models are consistent when conversion costs are not involved.

**Understocked Beginning State**

The next example is the case of a stand which is understocked in terms of both merchantable basal area and the relative amount of sawtimber basal area. A stand is assumed to contain only 35 square feet of merchantable basal area of which 35 percent is sawtimber basal area. Again, converting to a stand managed according to the results of the static optimal model (an MB-SRAT combination of 55-.55) is the long-term objective.

An example of what the diameter distributions of the beginning and ending states might be are illustrated in figure 7. The initial, understocked stand might have a maximum sized tree of only 12 inches and a Q of 1.24, while the desired ending structure might have a maximum sized tree of 16 inches and a Q of 1.27. Thus, the implied objective is to increase the basal area and maximum sized tree in the stand without
The optimal equilibrium steady-state MB-SRAT CC is 50-.45-5, rather than the combination 55-.55-5 derived from the static model. Thus, the two approaches are not consistent in their determination of the optimal steady-state management regime. The steady-state structure derived by the dynamic model is more theoretically correct than that derived by the static model because the dynamic programming model accounts for costs associated with the conversion process while the static model does not. In essence then, the dynamic programming model finds the optimal conversion strategy and optimal equilibrium steady-state simultaneously, rather than suboptimally by determining an optimal steady-state structure first and then finding an optimal conversion strategy separately given the specified steady state.

Overstocked Beginning State

The next example is the case of a stand which is overstocked compared to the static optimal regime in terms of both merchantable basal area and the relative amount of sawtimber basal area. A stand is assumed to contain 65 square feet of merchantable basal area of which 80 percent is sawtimber basal area. Although 65 square feet would not seem excessive in even-aged stands, it could be when managing uneven-aged loblolly-shortleaf pine stands. Again, converting to a stand managed according to the results of the static optimal model (an MB-SRAT combination of 55-.55) is the long-term objective.

An example of what the diameter distribution of the beginning and ending states might be are illustrated in fig. 9. The initial, overstocked stand might have a maximum sized tree of 24 inches and a Q of 1.15 while the desired ending structure might have a maximum sized tree of 16 inches and a Q of 1.27. Thus, the implied objective is to decrease the basal area and maximum sized tree and to increase the Q in the stand, that is, to reduce the relative sawtimber component of the stand.

Since the stand is overstocked, the dynamic programming model was specified to allow a stand to be harvested in the first year (fig. 10). Furthermore, rather than restricting the model to harvesting to a residual basal area of 55 square feet, the model was respecified to allow a stand to be cut to a residual of 40 square feet.

The first selection harvest takes place in the first year. All but 1 square foot is harvested from the sawtimber component. Approximately 3.4 MBF of sawtimber and 1.0 cord of pulpwood is cut in the first harvest. Even with the heavy sawtimber harvest, the residual stand has a high SRAT of .70. The reason for the high initial harvest is that the initial stand contains very little pulpwood basal area compared to sawtimber basal area. The only way to decrease SRAT from .80 to .55 is to decrease the sawtimber basal area since at least initially, the pulpwood basal area is fixed and must be allowed to grow.
It takes 92 years to convert the overstocked stand to an equilibrium steady-state structure (fig. 10), compared to only 30 years to convert the understocked stand. In the conversion period, MB after a harvest ranges from 40 to 50 square feet, while SRAT gradually decreases.

After the equilibrium steady-state regime at the end of 92 years is reached, MB is 50 square feet and SB is 25 square feet in that year and after all succeeding harvests; thus, the optimal SRAT is .50. At the end of every five years, the stand is harvested and yields 1.6 MBF of sawtimber and 0.9 cords of pulpwood.

Thus, the optimal equilibrium steady-state MB-SRAT-CC is 50-.50-5, rather than the combination 55-.55-5 derived from the static model. Again, the steady-state structure derived by the dynamic programming model is more theoretically correct than that derived by the static model since the dynamic programming model accounts for costs associated with the conversion process while the static model does not.

In general, it appears that given any constraint on the minimum MB allowed, the only way to decrease SRAT is to harvest sawtimber basal area while increasing the basal area in the pulpwod component. In so doing, the conversion from a stand with a high SRAT may require an initial heavy harvest in the sawtimber component, and possibly in total merchantable basal area, to the extent that the residual stand in the early years of conversion might be considered understocked. Unlike unmanaged stands that might be understocked, however, the conversion strategy for overstocked stands might involve understocked stands in which the structure can be controlled.

DISCUSSION

A dynamic programming model was developed for aiding in making long-term management decisions for uneven-aged loblolly-shortleaf pine stands in the Mid-South. The model was developed to determine optimal management strategies for converting initial stand structures to more desirable stand structures over a specified planning horizon given one of four long-term objectives: maximize total cubic foot production, maximize total sawtimber production, maximize total cash flow generated, or maximize the present net value of the cash flow generated. Although the model is a finite-horizon optimization model, given any specified initial stand structure, it can be used to simultaneously determine an optimal equilibrium steady-state regime and the optimal conversion strategy necessary to achieve it.

The objective used to illustrate the use of the model was to maximize the present net value of the cash flow generated over a planning horizon. Based on the model, a number of general conclusions can be made about the financial management of uneven-aged stands, at least of loblolly-shortleaf pine stands in the Mid-South. The first is that optimum management regimes derived from dynamic models will probably be different than those determined from static models. Those derived from static models will probably be suboptimal since static models do not consider the costs of converting existing initial stand structures to the structures recommended by the static models. On the other hand, dynamic models can simultaneously optimize the long-term equilibrium steady-state structure and the conversion to that structure.

An extension of the conclusion above is that the conversion of different initial structures might result in different long-term equilibrium steady-state structures. Based on the empirical analyses discussed previously, conversion of the understocked stand resulted in an equilibrium steady state MB-SRAT-CC of 50-.45-5. That is, at the end of every 5 years, a stand is cut back to
an MB of 50 square feet of which 45 percent consists of sawtimber basal area. Conversion of the overstocked stand, however, resulted in an equilibrium steady state of 50-55-5. The most probable reason that different beginning structures have different equilibrium steady-state structures is that conversion costs vary according to the structures of the initial stand structures. This conclusion implies two things: first, there is no single optimal steady-state structure that should be applied to all uneven-aged stands and second the conversion strategy, or management plan, for different uneven-aged stands should vary by the initial conditions of the stands if they are to be managed optimally over time.

The model can be used to determine the optimal management of both under- and overstocked stands. However, on a more practical basis, an understocked stand might be easier and take less time to convert to a more desirable stand structure or to an equilibrium steady state than it might take to convert an overstocked stand, particularly an overstocked stand with a high sawtimber component. In fact, as the example indicated above, the initial harvest of the overstocked stand might have to be so heavy that the residual stand would be considered understocked. Although understocked, however, the forest manager is able to control the structure (maximum diameter tree and diameter distribution) of the resultant understocked stand. This is not meant to imply that the management of understocked stands is preferable to the management of overstocked stands; indeed, the overstocked stands will more than likely be more profitable to manage. What this does mean, however, is that a greater amount of effort will probably be required in changing the structure of an overstocked stand and that the conversion process may take considerably longer.

LITERATURE CITED


