T I M B E R L A N D  I N V E S T M E N T  R E T U R N  
V O L A T I L I T Y  A N D  I M P L I C A T I O N S  F O R  
P O R T F O L I O  C O N S T R U C T I O N

Jon Caulfield, Vice President  
Wachovia Timberland Investment Management, Atlanta, GA

and

Ralph Meldahl, Assistant Professor  
Auburn University School of Forestry, Auburn, AL

A B S T R A C T

Traditional portfolio models which include timberland employ mean-variance analysis and assume return distributions are normal or that investor's utility functions are quadratic. With asymmetric distributions variance may inadequately measure risk because it assumes all volatility, even that coming from high returns, is undesirable. Also, many investors try to achieve a "target" return and are more concerned with avoiding target shortfalls than with the entire return distribution. Procedures dealing with non-normal distributions, or situations where variance is not an appropriate risk measure fall under the general heading of "downside risk".

Semivariance, a downside risk measure, was used to construct portfolios of southeastern timberland assets across three age classes and twenty geographic market areas. Initial analysis showed timberland investment returns were very positively skewed for young plantations. Skewness declined as the plantations matured. Semivariance resulted in more efficient portfolios than mean-variance analysis in that semivariance portfolios had lower risk for a given level of return. Semivariance also led to more positively skewed portfolios than mean-variance analysis and higher allocations of young growth timberland.

I N T R O D U C T I O N

A body of research appeared over the last decade which examines the return and risk characteristics of timberland and its role in investment portfolios. This interest started with the work of Mills and Hoover (1982) and includes research by Thomson (1987), Conroy and Miles (1987), Redmond and Cubbage (1988), Washburn and Binkley (1989) Deforest et al. (1991) and others.

These investigations employ portfolio theory and the Capital Asset Pricing Model (Markowitz 1959) and start with the
reasonable premise that investors attempt to achieve an optimal return-risk tradeoff from a feasible region of investment alternatives. The optimal tradeoff falls on an efficient frontier for which return is maximized for a given level of risk. Risk is measured by the variance, or equivalently, by the standard deviation of mean returns.

Washburn and Binkley (1989) describe several problems that arise when portfolio theory and the capital asset pricing model are applied to timberland investments. Among these are the: choice of an appropriate data series for measuring timber value changes; measurement of growing stock and bare land value; and the method by which periodic returns are calculated.

Classical portfolio theory approaches also assume that investment returns are normally distributed or that investor’s utility functions are quadratic. Under these circumstances, variance is an appropriate measure of risk.

But Harlow (1991) points out that while the concept of assembling portfolios with specific return-risk characteristics is clearly defined, a universally meaningful definition of risk is more elusive. For example, some managers consider risk to be the probability of returns falling below some benchmark, or "target" rate. Others may be more concerned with the magnitude of a potential loss. In each case, return variance may not be a wholly satisfactory risk measure.

Mean-variance analysis may also not provide the most efficient portfolio when return distributions are markedly skewed or where high returns are associated with high variances. Normal distributions, defined completely by the mean and variance, have no skewness. With skewed distributions alternative risk measures, falling under the general heading of "downside risk" (Harlow 1991) may be more appropriate.

This research focuses on one downside risk measure -- semivariance analysis -- and explores its applicability for constructing timberland portfolios. We begin by discussing traditional mean-variance versus alternative risk models. Sources of return from timberland investments are considered, and evidence is presented for a specific investment strategy which suggests that timberland returns tend to be positively skewed. Finally, implications for asset allocation using semivariance are discussed.

**MEAN-VARIANCE AND DOWNSIDE RISK MODELS**

The traditional mean-variance (E-V) formulation described by Markowitz (1959) minimizes the risk of a portfolio of risky
assets subject to a return constraint:

\[
(1) \quad \min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \rho_{ij} \sigma_i \sigma_j
\]

subject to:

\[
E(R_p) = \sum_{i=1}^{n} x_i E(R_i) = R^*
\]

\[
\sum_{i=1}^{n} x_i = 1
\]

where:

\[
E(R_p) = \text{expected portfolio return}
\]

\[
\sigma_p^2 = \text{portfolio variance}
\]

\[
x_i = \text{proportion of funds invested in asset } i
\]

\[
\rho_{ij} = \text{correlation between securities } i \text{ and } j
\]

\[
\sigma_i = \text{standard deviation of security } i
\]

Equation (1) is solved using quadratic programming to trace out the familiar efficient frontier in return-risk space (figure 1). When assets having low or negative correlations to one another are combined, portfolio variance for a given level of return is decreased relative to a non-diversified portfolio. When return distributions are normal or when investor’s utility functions are quadratic, the E-V formulation is a powerful tool for asset allocation.

Situations arise where measuring risk relative to the mean presents problems -- or at least, ambiguities. Figure 2 (Sortino and van der Meer 1991) shows hypothetical returns from assets C and D over time. The assets are perfectly negatively correlated, so portfolio variance is completely eliminated by investing equally in each asset. But it is likely that most investors would prefer a 100% investment in asset C, since despite its variability, returns are always higher than for D.

Figure 3 (Sortino and van der Meer 1991) describes a variation of the above: The investor has a minimum acceptable return of 8%. Asset A is highly variable but everywhere exceeds 8%. Assets B and C have perfect negative correlation and when combined result in a zero variance portfolio, but with an expected return lower than 8%. Traditional E-V asset analysis does not provide a useful solution.

Several approaches exist to deal with these limitations of E-V analysis. The most general is stochastic dominance analysis
FIGURE 1

Classical Mean-Variance Efficient Frontier
Target rate of return of 8% for an investor with Asset A dominates Assets B and C as well as "riskless" portfolio D for an investor with Asset A.

SDA uses pairwise comparisons of probability distribution of outcomes to eliminate inefficient distributions from consideration. It departs from E-V approaches in that it compares entire distributions of returns rather than a finite number of moments. Applying SDA to the examples in figures 2 and 3 would result in selecting investments C and A, respectively.

SDA is a powerful tool because no a priori assumptions are needed about the shape of the distributions. SDA also employs only very general assumptions about investor preferences. The technique is most useful for comparing discrete alternatives. However, no algorithm exists in SDA to combine assets into efficient portfolios (Nawrocki 1992). Therefore, while the technique can be applied to screen individual or portfolios of assets, it offers no assistance in asset allocation.

Semivariance analysis ($S^2$) (Markowitz 1959) overcomes some shortcomings of SDA. It is an asymmetric risk measure focusing on the probability of returns below a specified target return level. It is well-suited to dealing with skewed distributions and investors who do not have quadratic utility functions (Nawrocki 1992).

Semivariance is defined as the average of the squared deviations between a target return, and observations falling below that target. For this reason it is sometimes referred to as "target" semivariance.

Mathematically, the semivariance of a random variable (return) $R$ in period $t$ with target return $h$ is given by:

\[
(2) \quad S^2_h = \frac{1}{m} \sum_{t=1}^{m} \max \{0, (h-R_t)\}^2
\]

In (2), $m$ is the number of periods used to calculate $S^2$. If return exceeds the target $h$, the $h-R_t$ term is negative and the maximum function returns a 0. Therefore only below-target returns ($R_t<h$) provide a positive deviation which is squared and added to the calculation of semivariance (Nawrocki 1990).

Using this definition, the right tail of the return distribution does not contribute to risk. Instead, the right tail is reflected in the mean of the distribution. Therefore, two distributions with identical semivariance but different means are not the same. The distribution with a higher mean will be more positively skewed (Harlow 1991).

Interestingly, Markowitz (1959) was the first to propose semivariance. He recognized that investors are frequently more
concerned with losses relative to a target return. However, at the time of his research, the computational costs associated with semivariance analysis led him to focus on a E-V approach.

Besides being useful intuitively, semivariance has considerable theoretical appeal. Porter (1974) showed that semivariance is consistent with stochastic dominance. Put another way, efficient portfolios by SDA are also efficient using semivariance. This is a powerful result, because semivariance analysis has the advantage of an available algorithm for determining portfolio allocations.

Semivariance is most useful when assets considered for portfolios have skewed returns distributions. In such cases it leads to more efficient choices by including the most positively skewed distributions in portfolios. When distributions are approximately normal, variance is an adequate measure because portfolios identified by the two methods will be virtually identical.

Recent applications employing semivariance includes work by Lewis (1990) which examines portfolios with options as an asset category. A study by Marmer and Ng (1993) extends this work to consider other derivative-based strategies. Harlow (1991) employs the technique in the context of a global equity and debt allocation. In each case, portfolio mixes derived using semivariance differed from those estimated using E-V analysis.

The problem to be solved using semivariance analysis is:

\[
(3) \quad \min \quad S_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j S_i S_j \rho_{ij}
\]

subject to:

\[
E(R_p) = \sum_{i=1}^{n} x_i E(R_i) = R^* \\
\sum_{i=1}^{n} x_i = 1
\]

where:

- \(E(R_p)\) = expected portfolio return
- \(S_i\) = semideviation of asset \(i\) from target return
- \(x_i\) = proportion of funds invested in asset \(i\)
- \(\rho_{ij}\) = correlation between securities \(i\) and \(j\)

The above is almost identical to the classical portfolio problem except that semivariance is minimized for each return
level rather than variance. Note that semideviation ($S_1$) is used to calculate portfolio semivariance rather than standard deviation. $S_1$ is the square root of semivariance. Solution of the above model traces out an efficient frontier in return-semideviation space.

**METHODOLOGY**

Quarterly investment returns for a series of hypothetical timberland investments were constructed for the 10-year period 1983 to 1993. This is consistent with the fact that many institutional timberland investment vehicles are funds with fixed lives ranging from 7 to 12 years. The strategy employed was to purchase timberland at the beginning of the investment period and hold until the end of that period. At that time timber and land are both sold.

Returns were estimated for each of 10 southern states (AL, AR, FL, GA, LA, MS, NC, SC, TX, VA). Stumpage prices used to calculate periodic timber values for pine pulpwood, chip-and-saw and sawtimber in each period were from Timber Mart South (TMS). Returns were estimated for each of the two TMS regions in each state.

Investments were assumed to be loblolly pine (*Pinus taeda*, L.) plantations on site index 65 land planted with 700 trees per acre for all regions except Florida. In Florida investments were assumed made in slash pine plantation (*P. elliottii*, L.), for the same site index and planting density. These conditions are representative of plantations purchased by institutional investors in the south. Product volumes by age were determined for loblolly pine using Hafley and Smith (1989) and for slash pine using the GAPPS model (Burgan et al. 1989).

Bare land prices in each state came from appraisal and transaction data provided by independent forestry consultants. Regression equations were estimated to determine land values for each timber market over time.

To evaluate whether the age of the timberland investment influences the skewness of the distribution of returns, it was assumed that plantations could be purchased at three different ages during a rotation. These are categorized as "Young", "Intermediate" and "Mature" Growth.

Young Growth refers stands from time of establishment to the age of first merchantability for pulpwood. Intermediate Growth includes plantations from age of first merchantability to the point where the stand consists of approximately 25% chip-and-saw and the remainder pulpwood. Beyond this point large sawtimber begins to develop in the stand. Mature Growth accounts for all older plantations, for which sawtimber constitutes an increasingly significant component. For the site index and
planting densities considered, ages range from approximately 0-10, 11-20 and +21 for Young, Intermediate and Mature Growth, respectively.

Returns for different combinations of purchase and sales ages were screened to select the most appropriate purchase ages. The objective was to determine which purchase-sales age combination in each age class provided the highest return by region.

The ages employed in the final analysis were 9, 14, and 20 years for Young, Intermediate and Mature Growth, respectively. For a 10-year holding period, returns are thus estimated until ages 19, 24 and 30.

Investment value for each age class and region in period i were determined by multiplying merchantable product volumes by appropriate product prices and adding to land value. Periodic return was calculated by dividing by the investment value in period i-1.

Estimating investment value for Young Growth from age of purchase to first merchantability had to be approached differently than for Intermediate and Mature Growth. This was done in two steps. At purchase, the value of bare land in each region was estimated using the regression equation described above. The value of premerchantable timber was estimated using data provided by independent appraisers and added to bare land value.

The second step involved determining the volume and value of Young Growth at time of first merchantability, based on projected volumes, current timber prices and bare land value. Volume at first merchantability was multiplied by stumpage price each quarter to estimate what a newly merchantable stand would be worth at that time. A compound rate of value growth from purchase date to first merchantability was calculated quarterly. Initial stand value was compounded quarter by quarter, from time of purchase, by this variable rate of value growth. Upon reaching merchantability, stands were valued as for Established and Mature Growth.

A total of 60, 40-quarter return series ("assets") were thus derived (3 age classes x 10 states x 2 regions per state). Means, variances and skewness were determined for each. In each age class, the top 10 average annualized returns were used to construct portfolios consisting of Young, Intermediate and Mature Growth forests in different TMS regions.

Only 10 assets were used in each age class to avoid potential problems with matrix invertibility problems. There should be more observations used to calculate statistical inputs than there are assets used in computing the variance-covariance (or in this case, the semivariance-semicovariance) matrix.
Portfolios were constructed using traditional E-V and semivariance ($S^2$) analysis. For the latter, semideviation efficient portfolios were derived for target returns of 0%, 6.68% and 10%. The lowest target identifies a situation where investors seek simply to avoid losses. The 6.68% target represents the annualized 90-day Treasury Bill rate over the 10-year study period, and is a proxy for the risk-free interest rate. The 10% target was included to examine the sensitivity of the results. All returns are nominal.

RESULTS

Table 1 shows annualized returns, standard deviation and skewness measures by age class averaged over investment regions. Young Growth returns were more positively skewed than Intermediate Growth, which in turn were more skewed than Mature Growth. Those distributions with skewness significantly different from zero are denoted by asterisks. The test for significant skewness is from Snedecor and Cochran (1967).

Of 20 TMS regions, 16 Young Growth, 14 Intermediate Growth and 7 Mature Growth were significantly, positively skewed. The cause (or causes) of this skewness is not readily apparent. It may result from the very rapid rate of volume growth that takes place in the younger age classes, which would decline as plantations mature. Alternatively, it may that the distributions of stumpage price changes for different products are positively skewed. Or it may be a combination of factors.

Further research will address the causes of the observed positive skewness, although such a breakdown is beyond the scope of this study. But as shall be seen, the positive skewness characteristics of investment returns for Young and Established Growth has implications for portfolio construction.

It is clear from table 1 that on average, Intermediate Growth realized the highest returns over the investment period for this strategy and moderate risk, while Young Growth had slightly lower returns and low risk. Mature Growth had the lowest average return and highest risk. These results are not surprising given that pulpwood prices are generally observed to have lower volatility than chip-and-saw, which in turn has lower volatility than large sawtimber.

Figure 4 shows the $S^2$ efficient frontiers obtained for the three target returns of 0%, 6.68% and 10%, traced out in return-semideviation space. Also shown are portfolios with equal timberland allocations in each age class by region, for the three target return levels. In each case the latter have lower returns for given risk than the optimized portfolios.

The characteristics of the $S^2$ frontiers are as expected. The
Table 1. Average Returns, Standard Deviations and Skewness by Timberland Investment Age Class

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Average Coefficient of Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young</td>
<td>11.93</td>
<td>10.90</td>
<td>1.503 (16)*</td>
</tr>
<tr>
<td>Intermediate</td>
<td>12.12</td>
<td>13.64</td>
<td>1.229 (13)</td>
</tr>
<tr>
<td>Mature</td>
<td>11.09</td>
<td>15.16</td>
<td>0.5133</td>
</tr>
</tbody>
</table>

* Number of Timber Mart South regions (of 20) with significant positive skewness at 5% significance level.

Table 2. Asset Allocations and Skewness of Mean-Variance and Target Semideviation Portfolios, Averaged Over All Portfolios

<table>
<thead>
<tr>
<th>Age Class</th>
<th>Mean-Variance</th>
<th>Target Portfolios:</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean-Variance</td>
<td>0%</td>
<td>6.68%</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Young</td>
<td>45</td>
<td>44</td>
<td>47</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>Intermediate</td>
<td>54</td>
<td>56</td>
<td>53</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>Mature</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Average Skewness</td>
<td>1.40</td>
<td>1.43</td>
<td>1.52</td>
<td>1.59</td>
<td></td>
</tr>
</tbody>
</table>
Semideviations Efficient Frontiers

0%, 6.68%, and 10% Target Returns
higher the target return, the greater the semideviation and the portfolio skewness. This occurs because increasing the target means a greater proportion of the return distribution lies to its left. This leads to a concomitant increasing probability of a below-target return. As table 2 shows, the higher the target return, the higher the average portfolio skewness. In the table skewness is averaged over all return levels, as are the allocations to Young, Intermediate and Mature Growth.

$S^2$ portfolios for each target return were also compared to the E-V portfolios. Theory suggests that when distributions are skewed, $S^2$ analysis may provide more efficient portfolios in that there is a lower risk exposure for given return (Harlow 1991).

To make the comparison in consistent units, semideviations for the E-V portfolios were calculated for each return level. Results are shown in figures 5-7. Returns and allocations by age class were averaged and summarized in table 2.

For each target return the $S^2$ portfolios resulted in lower semideviation at each level. As the figures show, the E-V frontiers were to the right of the $S^2$ frontiers for all but the highest return level.

Theory suggests that compared to a E-V frontier, $S^2$ portfolios will be more positively skewed. Table 2 shows this is the case. For each target the $S^2$ portfolios have more positive skewness than E-V portfolios, and this skewness increases with the target return.

Another interesting result is that for the two highest target returns, the allocation of Young Growth is greater than for the E-V portfolios. Given that $S^2$ analysis leads to more positively skewed portfolios, and that returns for Young Growth generally had the most positive skewness, this result was expected.

Finally, the allocation of Mature Growth for the E-V portfolio was only 1%, going to 0% for all $S^2$ target portfolios (table 2). While a low allocation was expected, these extremes were somewhat surprising. They are explained largely by the return-risk characteristics of Mature Growth. Specifically, in most TMS regions over the study period, returns were lower and volatility was higher for Mature Growth relative to the other two ages classes.

**SUMMARY AND CONCLUSIONS**

Variance is not always an appropriate risk measure in situations when return distributions are non-normal or when risk-averse investor's utility functions cannot reasonably be assumed to be quadratic. In such cases, asset allocation models relying on a mean-variance approach can give ambiguous or risk-
Efficient Frontiers
E-V and 0% Target Semideviation

Figure 5.
Efficient Frontiers
E-V and 6.68% Target Semideviation

Figure 6
Efficient Frontiers

E-V and 10% Target Semideviation

Figure 7
inefficient results, and risk measures such as semivariance may therefore be more appropriate.

A simple buy-and-hold investment strategy was examined that assumed timberland was purchased in different southern markets and combined into portfolios. Asset allocations were made in timberland identified as Young, Intermediate or Mature Growth southern pine plantations. These were assembled using both semivariance and traditional mean-variance analysis.

In general, returns from Young Growth investments were greater than Intermediate Growth, which in turn exceeded Mature Growth. When portfolios were constructed, higher levels of target returns led to more positively skewed portfolios and higher Young Growth allocations.

It was found that semivariance analysis led to more positively skewed portfolios than those constructed using a mean-variance approach. The semivariance portfolios also had higher Young Growth allocations. Finally, semivariance provided more efficient allocations than mean-variance portfolios in the sense that they resulted in lower risk for a given return level.

LITERATURE CITED


